

**Inverted Haavelmo Effects in a General Equilibrium Analysis of  
the Impact of Implementing the  
Scottish Variable Rate of Income Tax<sup>\*</sup>**

by

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## **Abstract**

The Scottish Parliament has the authority to make a balanced-budget expansion or contraction in public expenditure, funded by corresponding local changes in the basic rate of income tax of up to 3p in the pound through the Scottish Variable Rate of income tax. However, this has never, as yet, been used. In this paper we attempt to identify and quantify the impact on aggregate economic activity in Scotland of implementing these devolved fiscal powers. This is achieved through theoretical analysis and simulation using a Computable General Equilibrium (CGE) model for Scotland. This analysis generalises the conventional Keynesian model so that negative balanced-budget multipliers values are possible, reflecting a regional “inverted Haavelmo effect”. Key parameters determining the aggregate economic impact are the extent to which the Scottish Government create local amenities valuable to the Scottish population and the extent to which this is incorporated into local wage bargaining.

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## 1. INTRODUCTION

In 1998 the Scotland Act (1998) gave the Scottish Parliament the authority to make a balanced-budget expansion or contraction in public expenditure. This fiscal adjustment would be funded by corresponding local changes in the basic rate of income tax of up to 3p in the pound. Although this Scottish Variable Rate of income tax has never, as yet, been used, there has been an active debate about the funding of those expenditures controlled by the Scottish Parliament.<sup>1</sup> The Commission on Scottish Devolution (2009), chaired by Kenneth Calman, *inter alia* recommends that the Scottish Parliament be forced to choose an appropriate income tax rate that can vary from the rest of the UK by up to 10p in the pound. In the budget of 2010, the Coalition Government affirmed its commitment to implement these proposals.<sup>2</sup>

This paper attempts to identify and quantify the impact on aggregate economic activity in Scotland of using the existing devolved fiscal powers. This is achieved through using a Computable General Equilibrium (CGE) model for Scotland to extend the conventional Keynesian model so that negative balanced-budget multiplier values are possible. This , reflects a regional “inverted Haavelmo effect”.(Haavelmo, 1945; Knoester and van der Windt, 1987).

## 2. THE THEORETICAL MODEL

We adopt a long-run, open-economy model of the region which is in the spirit of the disaggregated approach developed by Layard *et al.* (1991, ch. 6) with imperfect competition in the regional labour market. The price of imports and the cost of capital are both set exogenously in perfectly integrated national/international markets with regional exports determined through conventional trade functions (Armington, 1969; Engle and Rogers, 1996). That is to say, whilst output is produced under conditions of perfect competition within the

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1 This debate includes discussion of the appropriateness of the Barnett Formula, the desirability of a more explicit needs-based funding formula and the arguments for greater fiscal autonomy. For a more detailed bibliography see Lecca *et al.* (2010b).

2 The Independent Commission on Funding and Finance for Wales (2010), chaired by Gerry Holtham, makes similar arguments. Other commentators argue for a move within Scotland towards full fiscal autonomy but such a radical change is outwith the scope of this paper.

region, it is not a perfect substitute for the output of other regions.<sup>3</sup> This permits adjustments in the price of the regional good relative to the price of extra-regional goods and allows variation in the regional real and nominal wage in the long run.

Production occurs under well-behaved, linear homogeneous production functions with two factors, capital and labour and households are assumed to be homogeneous. In this section and the next we focus on a comparative-static long-run analysis where equilibrium implies that both the regional capital stock and population are optimally adjusted. There is zero net investment and zero net migration in long-run equilibrium states. We adopt the Layard *et al* (1991) regional migration function, in which net migration is a positive function of the inter-regional relative real wage and employment rates.<sup>4</sup> Capital stock is fully adjusted when actual and desired capital stocks in all sectors are equal.

We assume that the additional expenditure to be financed by the Scottish Variable Rate has a positive demand-side impact on regional output through a variant of the standard balanced budget multiplier. However, these are likely to be accompanied by negative supply-side effects operating through the impact of higher income tax rates on the labour market. We begin with a more detailed specification of our analytical model.

Equation (1) is the zero net migration condition.<sup>5</sup> It identifies the set of changes in the post-tax real consumption wage,  $dw$ , the employment rate,  $de$ , and the proportionate rate of income tax,  $d\tau$ , for which the net migration is zero:

$$dw = w_e de - \beta d\tau \quad w_e \leq 0, \beta \geq 0 \quad (1)$$

where  $\beta$  is a parameter indicating the degree to which households value public, as against private, consumption. Equation (1) indicates that there is a negative relationship between the post-tax real wage and the employment rate: across zero net migration (long-run) equilibria.

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<sup>3</sup> Layard *et al* (1991, ch. 6) also assume competitive commodity markets in their exposition of disaggregated labour markets.

<sup>4</sup> This function has its roots in Harris and Todaro (1970) and has been widely employed elsewhere. See Lecca (2010b) for further references.

<sup>5</sup> Equations (1) to (5) are expressed as total differentials given the normalisation:  $w, W, N, cpi = 1, \tau = 0$  and  $n = e$ . The corresponding equations in level form are given in Lecca *et al*, (2010b).

However, equation (1) is rather unconventional in that it includes the term  $-\beta d\tau$  in an attempt to capture the effect on the migration decision of the locally financed amenity.

Where individuals attach no value to this amenity,  $\beta = 0$  and the standard formulation of the net migration condition applies, with the post-tax real consumption wage governing migration decisions. However if, as is emphasised in the literature on fiscal federalism, there is a positive amenity effect, then  $\beta > 0$ . For a given employment rate, the larger the value of  $\beta$ , the lower the post-tax real consumption wage required to preclude net outmigration. When  $\beta = 1$ , the potential migrant is indifferent between marginal changes in local public expenditure and private consumption so that in this case the pre-tax real consumption wage drives migration.<sup>6</sup> Where,  $\beta > 1$  there is a positive marginal preference for local public expenditure over private consumption.

The change in pre-tax nominal wage,  $dW$  is defined in equation (2):

$$dW = dcpi + dw + d\tau \quad (2)$$

where  $dcpi$  is the change in regional consumer price index, given as:

$$dcpi = cpi_w dW \quad 1 > cpi_w \geq 0 \quad (3)$$

Equation (3) expresses the change in the regional consumer price index as a function solely of the change in the regional nominal wage. This parsimonious specification is permitted by the import-price and cost-of-capital exogeneity assumptions, together with the linear homogeneous nature of production.

The change in labour demand is given as a function of the change in the nominal pre-tax wage and the tax rate, so that:

$$dn = n_w dW + n_\tau d\tau \quad n_w \leq 0, n_\tau \geq 0 \quad (4)$$

It is important to note that equation (4) represents a general equilibrium relationship, constructed on the basis of full income endogeneity. Labour demand is negatively related to the nominal pre-tax wage through competitiveness and factor substitution effects.<sup>7</sup> On the

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6 That is to say, the individual is indifferent between £1 marginal private consumption and the public good implications of paying £1 more in tax.

7 Silvestre (1990) describes this general equilibrium labour demand curve as the "full" demand curve for labour. No nominal inertia is implied by this formulation, which is a re-parameterised version of a specification in which

other hand, labour demand is a positive function of the tax rate as a reflection of the operation of the conventional Keynesian balanced budget multiplier, the differential import propensities of public and private consumption expenditure, and the greater labour intensity of public sector activity.

Finally, through the bargaining function, the change in the real consumption wage is positively related to the change in the regional employment rate (Layard *et al*, 1991):

$$dw = b_e de - \alpha \beta d\tau \quad b_e \geq 0 \quad (5)$$

In this formulation of the regional bargaining function, the local amenity generated by the expenditure is allowed to influence wage bargaining behaviour directly. The parameter  $\alpha$ , which takes a value between 0 and 1, reflects the extent to which the value of the amenity is taken into account in the wage bargaining process.

The possible amenity effects on the local bargained real wage appear to be neglected in the literature on fiscal federalism. This is partly attributable to that literature's typical presumption of competitive labour markets. In such a situation, since the amenity provision is exogenous to the individual worker, it is ignored in the individual's work/leisure choice, so that only the post-tax real consumption wage matters.<sup>8</sup> This corresponds to a situation where the value of  $\alpha$  is zero. However, in the bargaining context the public good externality will be internalised in so far as local unions cover a significant section of the labour force and act co-operatively. Given that the scale of the amenity (under the proposed form of the Scottish Variable Rate) is tied directly to income and therefore to the bargained wage, the value of  $\alpha$  will rise above zero.

### **3. THE THEORETICAL ANALYSIS OF THE SCOTTISH VARIABLE RATE AND OTHER FISCAL INNOVATIONS**

#### **3.1 Regional Bargained Real Wage and Flow Migration Equilibria**

Using equations (1)-(5), the change in the five endogenous variables  $dn$ ,  $de$ ,  $d\pi$ ,  $dw$  and  $dW$ ,

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labour demand is a declining function of the real product wage.

<sup>8</sup> The labour supply decision could, of course, be influenced by the amenity if, for example, the amenity were complementary to leisure.

can be determined, given the change in the value of the exogenous taxrate,  $d\tau$ . A key variable in the analysis is the change in regional competitiveness that accompanies the fiscal expansion. In the present model, this is identified by the change in the nominal pre-tax wage,  $dW$ . This, together with the change in the employment rate, is determined by the interaction of the migration and bargaining functions. Combining equations (1), (2) and (3) produces the expression:

$$dW = \frac{z_e}{1-cpi_w} de + \frac{1-\beta}{1-cpi_w} d\tau \quad (6)$$

If  $d\tau$  is set equal to zero, equation (6) gives the initial zero net migration (ZNM) function in the change in nominal pre-tax wage-employment rate space. This is represented by the curve  $Z_0$  in Figure 1, which has a slope equal to  $z_e/(1-cpi_w) < 0$  and passes through the origin.

Similarly, combining equations (2), (3) and (5) gives:

$$dW = \frac{b_e}{1-cpi_w} de + \frac{1-\alpha\beta}{1-cpi_w} d\tau \quad (7)$$

Again, setting  $d\tau$  to zero in equation (7) produces the initial bargained real wage function (BRW) with a slope equal to  $b_e/(1-cpi_w) > 0$  which also passes through the origin. This is curve  $B_0$  in Figure 1.

Equations (6) and (7) are sufficient to tie down the change in the employment rate and nominal wage,  $de$  and  $dW$ , that accompany the fiscal expansion,  $d\tau$ :

$$de = -\frac{\beta(1-\alpha)}{b_e - z_e} d\tau \leq 0 \quad (8)$$

and

$$dW = \frac{b_e(1-\beta) - z_e(1-\alpha\beta)}{(1-cpi_w)(b_e - z_e)} d\tau \quad (9)$$

It is useful to analyse these results diagrammatically using Figure 1. Where a local income tax increase of  $d\tau$  is levied, the ZNM function (equation 6) moves vertically by an amount equal to  $((1-\beta)/(1-cpi_w))d\tau$ . The tax increase also shifts the BRW function vertically, but by  $((1-\alpha\beta)/(1-cpi_w))d\tau$ . Note that the parameter restrictions imply that  $1-\alpha\beta \geq 1-\beta$  so that the BRW function cannot experience a smaller upward movement than the ZNM function. We consider the impact of the fiscal expansion under alternative assumptions about the labour market.

### 3.1.1 A single regional bargain: $\alpha = 1$

Where  $\alpha = 1$ , the amenity value of the public expenditure is fully reflected in the bargaining equation. This would apply where the whole workforce is covered by a single bargain. In this case the ZNM and the BRW functions both move vertically by the same amount. There is no change in the employment rate, ( $de=0$ ), so that the equilibrium lies on the  $dW$  axis, that is on the line AJOC in Figure 1. The equilibrium point along this line depends solely on the value of  $\beta$ .

Where  $\beta = 0$ , so the additional public expenditure produces no amenity value to local residents, the ZNM and the BRW functions both shift upwards by  $d\tau/(1-cpi_w)$  to  $B_1$  and  $Z_1$  respectively and the equilibrium is at A. The change in the pre-tax nominal wage is  $d\tau/(1-cpi_w)$  so that the full tax increase is incorporated into higher nominal wages, including an element  $(1-cpi_w)^{-1}$  to accommodate the increase in regional cpi. Alternatively, where  $\beta = 1$ , so that the value of the increased public expenditure to local residents just equals the forgone private consumption, neither the ZNM nor the BRW curve moves. There is simply a switching of part of the pre-tax wage from private to public expenditure: there is no change in the employment rate and no loss of competitiveness through higher nominal wages. The new equilibrium remains at the origin.

As the value of  $\beta$  varies between zero and one, the equilibrium moves between points A and the origin, 0. If the value of  $\beta$  is greater than unity, so that the residents have a positive preference for public as against private consumption, the ZNM and BRW functions move downwards so that the nominal pre-tax wage fall to an equilibrium at a point such as C.

### 3.1.2 Perfectly competitive labour market: $\alpha = 0$

Where the labour market is perfectly competitive,  $\alpha = 0$ . From equation (7) this means that in Figure 1, for any value of  $\beta$  the BRW curve moves upwards by the amount  $d\tau/(1-cpi_w)$  to  $B_1$ . The subsequent competitive labour market equilibrium will lie on this line, ADMLE. Where the public amenity has no value, so that  $\beta = 0$ , the new equilibrium is at A, with the change in



the nominal pre-tax wage as  $d\tau/(1-cpi_w)$ . Where  $\beta = 1$ , the ZNM curve remains static at  $Z_0$  and the new equilibrium is at D. Using equation (9) gives the result that at D, the change in the nominal pre-tax wage is positive and is given as:

$$dW = -\frac{z_e d\tau}{(1-cpi_w)(b_e - z_e)} > 0 \quad (10)$$

Values of  $\beta$  between zero and one generate equilibria along the line segment AD and values of  $\beta$  greater than 1 lead to equilibria further down the BRW function  $B_1$ , to points such as M, L and E.

### 3.1.3 The general case of imperfectly competitive labour market: $1 \geq \alpha \geq 0$

Where  $\alpha$  lies between zero and unity, for a particular value of  $\beta$  the associated values of  $dW$  and  $de$  lie on the appropriate ZNM line. For example,  $\beta = 1$ , the appropriate ZNM function is  $Z_0$ . The equilibrium will lie on the line 0HD, where the closer the value of  $\alpha$  is to zero, the closer is the equilibrium to D. For lower values of  $\beta$  ( $1 > \beta \geq 0$ ), the ZNM function is above and parallel to 0HD. The relevant range of equilibrium values will again lie between the vertical zero employment rate change line, AJ0C, and the  $B_1$  BRW function ADMLE. Again, the more competitive the labour market, the closer the equilibrium will be to the ADMLE curve.

It is clear that the equilibrium must lie in the shaded area in Figure 1. Where  $1 \geq \beta \geq 0$ , the equilibrium is within the darker shaded triangle, AD0. With these parameter restrictions, there is only one point where inter-regional competitiveness is not negatively affected by the fiscal expansion. This is where  $\alpha = \beta = 1$ , the equilibrium at the origin. In every other outcome in the triangle AD0, regional competitiveness is reduced. Where  $\beta > 1$  the possible equilibria are represented by the lighter shading. In these cases there are combinations of the BRW and ZNM functions where the nominal wage falls, so that regional competitiveness could increase with a local fiscal expansion.

### 3.1.4 Changes in employment, $dn$

The results in Figure 1 give changes in the nominal wage and the employment rate, but our central concern is changes in the level of economic activity and specifically changes in the

employment level. In general the employment level and the employment rate diverge because the population (and therefore the work force) is endogenous. Figure 1 shows that under a wide range of parameter values, a balanced fiscal expansion generates an increase in the nominal wage and therefore reduces regional competitiveness. However, where this is the case, the change in employment is the result of the trade-off between the positive demand side stimulus, generated by the Keynesian balanced budget multiplier, and the potential negative competitiveness effects, produced by the higher nominal wage.

This analysis follows that of Knoester and van der Windt (1987) who argue that, at a national level, forward tax shifting by workers produces a reduction in competitiveness and therefore a possible inverted Haavelmo effect; that is, a negative balanced budget multiplier. Substituting equation (9) into equation (4) gives the employment change as;

$$dn = \left[ \frac{n_\tau(1 - cpi_w)(b_e - z_e) + n_w(b_e(1 - \beta) - z_e(1 - \alpha\beta))}{(1 - cpi_w)(b_e - z_e)} \right] d\tau \quad (11)$$

where  $\frac{\partial(dn)}{\partial\alpha}$ ,  $\frac{\partial(dn)}{\partial\beta}$ ,  $\frac{\partial^2(dn)}{\partial\alpha\partial\beta} \geq 0$ .

Clearly the change in employment is positively related to the value of the amenity generated by the government expenditure,  $\beta$ , and the extent to which this is reflected in the regional bargained wage,  $\alpha$ . However, our central concern is the sign of the employment change that accompanies a balanced fiscal expansion. Again we approach this both diagrammatically and algebraically.

First, setting  $dn$  equal to zero in equation (4) and rearranging gives the value for  $dW$  for which the fiscal expansion has a zero employment impact:

$$dW = - \frac{n_\tau}{n_w} d\tau \geq 0 \quad (12)$$

This line is plotted in Figure 1 as GMHJK, where the intercept J on the  $dW$  axis is  $-\frac{n_\tau}{n_w} d\tau$ .

All combinations of the change in pre-tax nominal wage and employment rate below GHJK produce an increase in employment.

Equilibria involving no increase in the pre-tax nominal wage are unambiguously associated with an expansion in employment. This includes the origin, which is the equilibrium where  $\alpha = \beta = 1$ . Here no price changes accompany the fiscal expansion so that the regional economy operates as under the standard Keynesian balanced budget multiplier with  $dn = n_\tau d\tau$ . But there is also a range of equilibria where the change in pre-tax nominal wage is positive, so that regional competitiveness falls but employment still rises. The corollary is that as long as there is a positive demand side stimulus from the balanced fiscal expansion, so that  $n_\tau > 0$ , there is always some set of values for  $\alpha$  and  $\beta$  in the range  $1 \geq \alpha, \beta \geq 0$  where employment change will be positive. In Figure 1 the equilibria falling in the triangle OHJ are in this category.

An alternative approach is to set  $dn$  equal to zero in equation (11), and rearrange to generate the combinations of the parameters  $\alpha$  and  $\beta$  that produce zero employment change. For a positive employment change the  $\beta$  parameter must be greater than a minimum value,  $\underline{\beta}$ , given by:

$$\beta > \underline{\beta} = \frac{b_e - z_e}{b_e - z_e \alpha} \left[ 1 + \frac{n_\tau (1 - cpi_w)}{n_w} \right] \quad (13)$$

where  $\frac{\partial \beta}{\partial \alpha} < 0$  and  $\frac{\partial^2 \beta}{\partial \alpha^2} > 0$ .

Equation (13) is represented schematically in Figure 2. In this diagram the values of  $\alpha$  lies within the range  $\alpha \in [0,1]$  and whilst in principle  $\beta$  can take any value, we confine our attention to positive values and are particularly interested in range  $\beta \in [0,1]$ .<sup>9</sup> Combinations of  $\alpha$  and  $\beta$  above the zero employment change locus, LNM, generate an increase in employment, whilst those below this line are associated with employment decline. We know from equation (11) that where  $\alpha = \beta = 1$  the employment change associated with the fiscal expansion is positive. This is given as point P in Figure 2. The zero employment change (ZEC) locus therefore lies below this point but its exact position depends upon the general equilibrium elasticities  $n_w, n_\tau$  and  $cpi_w$ , together with the bargaining and migration parameters  $b_e$  and  $w_e$ .

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9. A negative  $\beta$  value indicates a publically generated local amenity that has negative value for residents and potential migrants. Whilst it is possible to think of examples of such local public expenditure, these are ruled out in this analysis.

The partial derivatives from equation (13) show that the ZEC locus in Figure 2 is downward sloping and convex. We are interested additionally in the values of  $\beta$  at the end points; that is, where  $\alpha$  is zero and one, identified as  $\beta^0$  and  $\beta^1$  respectively. There are three interesting general cases, which are specified by the value of the ratio of the two general equilibrium employment elasticities,  $n_\tau/n_w$ .

First, if  $\frac{n_\tau}{n_w} \geq -\frac{1}{(1-cpi_w)}$ , then  $\beta^0, \beta^1 < 0$ . Employment increases for all positive values of  $\alpha$

and  $\beta$ . In Figure 1, this corresponds to the situation where the horizontal zero employment line, there shown as GMHJK, lies above the point A, and in Figure 2 the ZEC locus lies below the origin and therefore outwith the positive quadrant.

Second, if  $-\frac{1}{(1-cpi_w)} > \frac{n_\tau}{n_w} > \frac{z_e}{(1-cpi_w)(b_e-z_e)}$  then  $\beta^0, \beta^1 \in [0,1]$ , with  $\beta^0 > \beta^1$ . In Figure 1

this is where the horizontal zero employment change line lies between points A and D.

Third, if  $\frac{z_e}{(1-cpi_w)(b_e-z_e)} > \frac{n_\tau}{n_w}$ , the value of  $\beta^0 > 1$  and  $\beta^1 \in [0,1]$ . This is the situation

depicted in Figure 2 where the point L is  $(0, \beta^0)$  and M is  $(1, \beta^1)$ . It also corresponds to the outcome represented in Figure 1, where the GMHJK zero employment change line lies between points D and L.

### 3.2 National Wage Bargaining with Regional Flow Migration Equilibrium

In Section 3.1 we have adopted a local real wage bargaining framework for the determination of the regional wage. However, it is often argued that within the UK the regional wage is set at the national level, either by national bargaining or through company-wide wage setting in multi-plant firms. In this case, equation (5) can be replaced by  $dW=0$ . The appropriate substitutions generate the results:  $dcpi=0$ ,  $dn=n_\tau d\tau > 0$ ,  $dw=-d\tau < 0$  and

$de = -\frac{(1-\beta)}{z_e} d\tau > 0$  iff  $\beta < 1$ . Employment increases. However the real wage falls by the full

amount of the tax change and employment rate rises for values of  $\beta$  less than unity, in order to

satisfy the zero net migration constraint. Essentially, with national wage bargaining there are the familiar expansionary demand effects associated with the shift from private consumption to public expenditure, but no adverse competitiveness impacts.

In terms of Figure 1, bargaining function,  $B_0$ , can be replaced by a zero-change pre-tax nominal wage line, which is the de axis, line  $OL$ . The equilibrium is now where the zero net migration function cuts the de axis. The equilibrium value of  $dW$  is clearly zero but, as argued above,  $de$  depends on the value of  $\beta$ . Where  $\beta < 1$ , the equilibrium lies to the right of the origin so that  $de > 0$ . Where  $\beta > 1$ , the equilibrium is to the left of the origin and  $de < 0$ .

### **3.3 Regional Real Wage Bargaining and Zero Labour Mobility Equilibria**

Inter-regional migration has played a central role in the analysis up to this point. However, UK regional problems are often linked to restrictions in geographic labour mobility. It is therefore valuable to investigate the consequences of imposing the limiting case of zero labour mobility. Lecca *et al* (2010b) compares in depth the long-run outcomes following a fiscal expansion under zero labour mobility and flow equilibrium migration. We give an intuitive account here.

First, if under zero labour mobility employment increases, employment expansion is even greater under flow equilibrium migration. This is because in this case, the real wage and employment rate must both increase under zero labour mobility. Therefore if flow equilibrium migration is introduced, the subsequent immigration eases the pressure in the local labour market, reducing the nominal wage thereby increasing competitiveness and employment.

Where employment falls under zero labour mobility the situation is a little more complex. The bargained real wage curve must have shifted upwards so that  $\alpha$  and/or  $\beta < 1$ . If the value of  $\alpha$  is low, it could be that whilst the employment rate has fallen, there is a sufficient increase in the real wage, taking into account the value of the new publically produced amenity, for there to be immigration. This would mean that under flow migration the fall in employment would be mitigated or could even be positive. However, if the  $\beta$  is low, the reduction in the employment rate will lead to outmigration and the employment decline will be exacerbated.

## **4. COMPUTABLE GENERAL EQUILIBRIUM MODELLING WITH A SCOTTISH MODEL**

### **4.1 Regional Computable General Equilibrium Modelling**

The use of CGE models to identify the likely impacts of fiscal innovations is well established both at the national (e.g. Shoven and Whalley, 1992) and regional (e.g. Hirte, 1998; Partridge and Rickman, 1998) levels. In this case, CGE analysis is particularly appropriate for a number of reasons. First, it is clear from the analysis in the previous subsections that the key general equilibrium elasticities determine not just the quantitative but also the qualitative characteristics of the balanced fiscal expansion equilibrium. Such elasticities are difficult to determine without general equilibrium simulation. Second, the analytical model gives only long-run equilibrium values: it tells us nothing about the time path to this equilibrium. Third, a CGE model gives the change in values for a wide range of aggregate variables and allows for sectoral disaggregation.

One problem in tackling this issue through simulation is that existing UK empirical work offers no direct evidence on the parameter values  $\alpha$  and  $\beta$  since the UK has no experience of a local income tax. Furthermore, the evidence from other countries on the values of  $\alpha$  and  $\beta$  is mixed and appears to depend on the composition of public expenditures.<sup>10</sup> However, our reading of the literature is that the tendency of conventional neoclassical analysis to ignore the potentially beneficial impacts of regional public expenditures is rejected by those studies that provide a balanced treatment of tax and expenditure effects (e.g. Gabe and Bell, 2004). Furthermore, the suggestion that the composition of expenditures influences the values of key parameters implies that they are sensitive to policy choices. Against this background, there is a strong case for progressing the analysis via numerical simulation as long as the sensitivity of the results to the values taken for  $\alpha$  and  $\beta$  is a central feature.

### **4.2 AMOS: A macro-micro model of Scotland**

<sup>10</sup>For a short set of references see Lecca (2010b).

AMOS is a CGE modelling framework parameterised on data from Scotland.<sup>11</sup> Essentially, it is a fully specified, empirical implementation of the skeletal theoretical model developed in Sections 2 and 3. It has three domestic transactor groups, namely the personal sector, corporations and government; and four major components of final demand: consumption, investment, government expenditure and exports. There are eleven commodities/activities but in the simulation results reported in Tables 1, 2 and 3, these are aggregated into three broad industrial groups: manufacturing, non-manufacturing traded and a sheltered sector.

In this version of the model, consumption and investment decisions reflect intertemporal optimization with perfect foresight (Lecca *et al*, 2010a,b). Real government expenditure is equal to the base year level plus an additional amount that just exhausts the increment to tax revenue raised by the local income tax. This implies that government expenditure becomes dependent on the entire general equilibrium of the system, which is exactly what would happen if the Scottish Variable Rate were to be implemented. The demand for Scottish Rest of the UK (RUK) and Rest of the World (ROW) exports is determined via conventional export demand functions where the price elasticity of demand is set at 2.0. Imports are obtained through an Armington link (Armington, 1969) and therefore relative-price sensitive with trade substitution elasticities of 2.0 (Gibson, 1990).

In all the simulations in this paper we impose a single Scottish labour market characterised by perfect sectoral mobility. All sectors are taken to be perfectly competitive and produce using multi-level CES production functions with elasticities of substitution of 0.3 (Harris, 1989). We do not explicitly model financial flows, our assumption being that Scotland is a price-taker in competitive UK financial markets.

As regards demographic developments, we assume no natural population change but in the default version of the model, the labour force adjusts using the econometrically parameterised regional net migration function reported in Layard *et al* (1991), augmented to accommodate the amenity effects discussed in Sections 2 and 3. The model starts in long-run equilibrium with zero net migration flow and, in any subsequent period, migration is positively related to

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<sup>11</sup> AMOS is an acronym for *A Macro-micro Model Of Scotland*. The model is calibrated using a Social Accounting Matrix based around the 2004 Scottish Input-Output Tables (Scottish Government 2007).

the gap between regional and national real tax-adjusted wages, and negatively related to the gap between national, and regional unemployment rates:

$$m = \varsigma - 0.08[\ln(u^S) - \ln(u^R)] + 0.06 \left[ \ln\left(\frac{w^S}{cpi^S}\right) - \beta \ln(1-\tau) - \ln\left(\frac{w^R}{cpi^R}\right) \right] \quad (16)$$

where  $m$  is net in-migration as a proportion of the regional population;  $u$  is the unemployment rate for Scotland,  $\beta$  is the relative valuation of the public expenditure and the S and R superscripts stand for Scotland and the Rest of the world, respectively.<sup>12</sup> In the long run, there is an implied zero-net-migration condition that yields estimates of the optimal spatial distribution of population. This is:

$$\ln\left(\frac{w^S}{cpi^S}\right) = b + 1.33 \ln(u^S) + \beta \ln(1-\tau) \quad (17)$$

where  $b$  again is a calibrated parameter. Wage setting is determined by a regional bargained real wage function that embodies the econometrically derived specification given in Layard *et al* (1991), again augmented by amenity effects:

$$\ln\left(\frac{w^S}{cpi^S}\right) = c - 0.113 \ln(u^S) + \alpha \beta \ln(1-\tau) \quad (18)$$

where  $\alpha$  represents the extent to which the amenity effect is reflected in the wage bargain and  $c$  is a calibrated parameter.

## 5. SIMULATION RESULTS

We use AMOS to simulate the long-run effects of the Scottish Variable Rate on the Scottish economy. Given that the model is parameterised on 2004 data we use the 2004 HM Treasury Budget estimate that these fiscal powers would raise £810 million at 2004 prices, which represents a 1.52 percentage point rise in average personal income tax in AMOS.

### 5.1 Inter-regional migration and regional bargaining

Table 1 reports the long-run proportionate changes in Scottish employment after the introduction of such a tax for combinations of  $\alpha$  and  $\beta$ , where  $\alpha \in [0,1]$ ,  $\beta \in [0,2]$ . Figure 3

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<sup>12</sup> Equation (18) is estimated in terms of relative unemployment rates but these are just the inverse of the employment rate used in Sections 2 and 3.



illustrates these results graphically. These outcomes are consistent with the analytical results generate in Section 3.4. When the parameters  $\alpha = \beta = 1$ , the employment change is positive at 0.68%, replicating the standard Keynesian balanced budget multiplier, and the level of employment change is positively related to the values of the parameters  $\alpha$  and  $\beta$ . Even a relatively small reduction in the value of either of these parameters below unity leads to employment falling with a balanced budget fiscal expansion.

Clearly, for a fiscal expansion to have a positive impact on employment, existing and potential Scottish residents must value the resulting amenity relatively highly. Where  $\alpha = 1$ , so that the value of the consumption of the public good is fully reflected in the wage bargain, employment falls with a fiscal expansion for values of  $\beta$  less than 0.76: for  $\alpha = 0.8$ , employment falls for values of  $\beta$  less than 0.93.

However, equally, an increase in public expenditure that has a high marginal value for existing and potential Scottish residents has a depressing impact on aggregate long-run economic activity if these benefits are not incorporated into the wage bargain. Note that where the labour market is perfectly competitive ( $\alpha = 0$ ), employment always falls with the range of values for  $\beta$  given here. Where  $\beta = 1$ , if  $\alpha$  takes a value below 0.74, employment falls. This is represented by point H in Figure 1 and N in Figure 2. Again, where  $\beta = 0.8$ , employment falls for any value of  $\alpha$  below 0.95.

In Table 2 we give the proportionate changes in a more comprehensive set of economic variables for four particular combinations of  $\alpha$  and  $\beta$ . This allows a fuller investigation of the economic forces at work in each of these cases. The first column reports results from the simulation where  $\alpha = \beta = 1$ , the conventional Keynesian balanced budget multiplier case, albeit with endogenous investment and population.

In this simulation there is no change in either the pre-tax nominal wage or the unemployment rate, so that there are no adjustments in long-run value-added prices, the cost-minimising choice of production technique in each industry or the level of exports. Essentially the economy operates as an extended input-output system where, in each sector, output,

employment and capital stock vary by the same proportionate amount.<sup>13</sup> The demand disturbance comes through the switching of private consumption expenditure to public expenditure.

As argued in Sections 2 and 3, this has a general expansionary impact on the regional economy. The 3.77% increase in government expenditure produces an increase in Scottish GDP of 0.47% and in employment and population of 0.68%. However, the adjustment in consumption and government demand has an uneven effect across sectors. Value added in the sheltered sector, which is most strongly represented in government expenditure, increases by 2.26%. In the other two sectors value added falls, but by a relatively small amount, -0.02%, in manufacturing and by -0.26% in non-manufacturing traded.

The second column gives the simulation results with the parameter values  $\alpha = 0$  and  $\beta = 1$ . This simulation corresponds to the equilibrium represented by point D in Figure 1. This combination of parameter values generates increases in the nominal pre-tax wage (2.39%) and the unemployment rate (1.35%), resulting in negative competitive effects as value-added prices increase in all sectors. The fall in exports swamps any expansionary impacts generated by the other shifts in final demand. Scottish GDP, total employment and population decline by 1.97%, 1.95% and 1.71% respectively, with activity falling in all sectors, though particularly the non-sheltered sectors. The fall in Scottish real income is associated with a smaller rise in the endogenous public expenditure, which increases by 1.69% with these parameter values.

The results in the third column, derived where  $\beta = 0$ , correspond to point A in Figure 1, where the amenity has no value to existing or potential Scottish residents. In this simulation there is no change in the post-tax real consumption wage or the unemployment rate. The nominal pre-tax wage increases by 2.60%, the full extent of the tax plus the rise in the consumer price index. This results impact on individual sectors is qualitatively similar to the case where  $\alpha = 0$  and  $\beta = 1$ , except that the results here are more extreme. This is the "worst-case" scenario for the Scottish Variable Rate.

The final simulation, reported in column four, adopts the parameter values  $\alpha = 0.80$ ,  $\beta = 1.20$

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<sup>13</sup> The figures reported in Table 2 do not show this precisely because the results given for the three sectors are for aggregations of the 11 sectors in the model. For more sectoral detail see Lecca *et al.*, (2010b).

and represents an equilibrium lying in the area OHML in Figure 1 where  $\beta > 1$  and both employment and the nominal pre-tax wage increase. The 0.05% rise in the pre-tax nominal wage following the introduction of the Scottish Variable Rate reduces exports in all sectors. However, the other expansionary fiscal demand impacts produce a more than offsetting effect on overall Scottish aggregate activity. Therefore, although there are small employment falls in the manufacturing and non-manufacturing traded sectors of 0.08% and 0.35%, employment in the sheltered sector rises by 2.20% producing an aggregate increase in GDP, total employment and population.

## **5.2 Inter-regional migration and national bargaining**

The results presented in the first column of Table 2 also give the outcome where there is national bargaining with the value of  $\beta = 1$ . The nominal wage and unemployment rate remain unchanged, and population rises by 0.68%. There is a positive stimulus to the Scottish economy that corresponds to the conventional Keynesian balanced budget multiplier. For alternative values of  $\beta$  under national bargaining, the changes in the unemployment rate and population level do vary so as to maintain the zero net migration requirement. But in the present parameterisation of the model, this has no direct impacts on household expenditure, so that the change in all other variables is as under column one of Table 2. Where  $\beta$  equals zero, so that the additional public expenditure has no value to Scottish residents, population rises by 0.42% and the unemployment rate falls by 1.45%. On the other hand, where  $\beta = 1.2$ , population increases by 0.73% and the unemployment rate rises by 0.29%.

## **5.3 Zero population mobility and regional bargaining**

Table 3 shows the percentage changes in key variables for the fiscal expansion in a model with bargaining and zero population mobility. As discussed in Section 3.3, in this case the migration function is dropped and the bargaining function is affected by the product of the two parameters:  $\alpha\beta$ . This composite parameter measures the extent to which the increase in public expenditure is incorporated into reduced wage claims.

The first column gives results where  $\alpha\beta = 1$ , which would correspond, *inter alia*, to the set of

values  $\alpha = 1$ ,  $\beta = 1$ . Here GDP increases by 0.17%, employment by 0.35% and the nominal pre-tax wage by 0.30%. Increased nominal wages leads to higher value added prices and exports fall in all sectors. Value added increases in the Sheltered sector but falls in the Manufacturing and Non-M anufacturing sectors.

The simulation results for  $\alpha = 0.8$ ,  $\beta = 1.2$  (so that  $\alpha\beta = 0.96$ ) again produces a stimulus to GDP but here slightly lower with a slightly greater loss in competitiveness. Finally where  $\alpha\beta = 0$ , there is a decline in GDP, and employment by 1.22% and 1.14%. This corresponds to the simulations previously where  $\beta = 0$  or where  $\alpha = 0$ ,  $\beta = 1$ . In this case there are large reductions in exports in all sectors and although value added in the Sheltered sector rises by 0.63%, it falls in the M anufacturing and Non-Manufacturing Traded sectors by 1.69% and 2%.

Table 4 gives the percentage change in employment for combinations of the parameters  $\alpha$  and  $\beta$  with zero population mobility. Compare these results with those in Table 1 for the flow migration model. First, note that for values of  $\alpha\beta \geq 0.8$ , where the employment change is positive for the model with zero population mobility, the employment change is greater for the model incorporating inter-regional migration. There is one set of parameters ( $\alpha = 0.4$ ,  $\beta = 1.8$ ) where the employment change under zero population mobility is negative, but under inter-regional migration it is positive. There is also one set of parameters, where  $\alpha = 0.6$ ,  $\beta = 1.2$ , where employment change in both models is negative, but the zero population mobility model has a larger negative value. Both cases are on the same bargaining function where  $\alpha\beta = 0.72$ . In all other case, where the value of  $\alpha\beta \leq 0.64$ , the employment falls in both models and the fall is greater where there is inter-regional migration.

## **6. THE ADJUSTMENT PROCESS AND SENSITIVITY**

### **6.1 Time Period of Adjustment**

Figure 5a and 5b plot the period-by-period percentage changes for the Tobin's  $q$ , disaggregated by sector, associated with the introduction of the Scottish Variable Rate. Figure 5a is for the simulation where  $\alpha$  and  $\beta$  are both unity and Figure 5b where both are zero. The adjustment process, which depends on the interaction of migration, consumption and

investment decisions, depends on these parameter values. In Figure 5a the adjustment is complete within around 10 periods, while in Figure 5b it takes almost 20 periods for the Tobin's  $q$  to achieve the constant accumulation rate. A more detailed discussion is available in Lecca *et al* (2010).

## 6.2 Sensitivity Analysis

One criticism of CGE models is that they are not fully econometrically estimated and the results might be very sensitive to imposed parameter values. In this sensitivity exercise, we following the method used by Harrison and Vinod (1992). 1000 simulations are run in which the value of the constant elasticity of marginal utility is selected from a uniform distribution with the range (0.2 – 1.6), the value-added production function substitution elasticities from the range (0.1 - 0.5) and the trade elasticities from the range (0.1 - 4.0). We divide each distribution into 4 equal intervals and adopt a complete randomized factorial design. Each of the 1000 simulations is run for 50 periods.

Figures 6 and 7 report the results of systematic sensitivity analysis on the period-by-period simulations for two of the  $(\alpha, \beta)$  combinations reported in Table 2: (1,1) and (0.8, 1.2). In each period the graphs show the mean solution value of the percentage increase in total employment of the 1000 simulations together with the plus-or-minus-one-standard-deviation range of results. In general, the one standard deviation confidence limits are small and fall over time. This is because in these two cases, migration and investment reduce the price deviations upon which the production and demand elasticities bite. This is particularly apparent in the simulation results reported in Figure 6, where  $\alpha$  and  $\beta$  are both unity. In this simulation extended Input-Output results hold in the long run in which there are no relative price changes, so that variation in price elasticities play no role and the confidence range ultimately collapses to a single point (McGregor *et al*, 1996). In Figure 7, where  $(\alpha, \beta)$  values are (0.8, 1.2), price changes are still present in the long run so that employment remains sensitive to these parameter values.

## 7. CONCLUSIONS

A key result of this analysis is that the impact of a balanced fiscal expansion on regional economic activity depends crucially on the value that existing and potential residents place on the resulting public amenity and the extent to which this value is reflected in a moderation of local pay claims. Such moderation is not available in a perfectly competitive labour market where there is no mechanism to internalise the benefit of the publicly provided externality. Our simulation results suggest that the balanced budget employment multipliers would be negative in such circumstances. However, in an imperfectly competitive labour market, where unions are concerned with the general welfare of their members, or where the nominal wage is set exogenously, such inverted Haavelmo effects do not necessarily occur or might be offset by the positive demand effects. Under these circumstances there could be significant potential welfare benefits to Scotland from the use of this fiscal innovation. Up to now, the Scottish Government have not used the fiscal powers embodied in the Scottish Variable Rate. However, if the recommendations of the Calman Commission are introduced, the probability that such powers will be used in the future is much increased.

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**Table 1. Long-run % change in employment after the introduction of the Scottish Variable Rate with endogenous population**

		$\alpha$					
		0.00	0.20	0.40	0.60	0.80	1.00
$\beta$	2.00	-1.73	-0.68	0.37	1.44	2.51	3.59
	1.80	-1.77	-0.83	0.12	1.07	2.03	3.00
	1.60	-1.82	-0.98	-0.14	0.71	1.56	2.42
	1.40	-1.86	-1.13	-0.39	0.34	1.09	1.83
	1.20	-1.90	-1.28	-0.65	-0.02	0.62	1.25
	1.00	-1.95	-1.43	-0.90	-0.38	0.15	0.68
	0.80	-1.99	-1.58	-1.16	-0.74	-0.32	0.10
	0.60	-2.04	-1.72	-1.41	-1.10	-0.78	-0.47
	0.40	-2.08	-1.87	-1.66	-1.45	-1.25	-1.04
	0.20	-2.12	-2.02	-1.92	-1.81	-1.71	-1.60
	0.00	-2.17	-2.17	-2.17	-2.17	-2.17	-2.17

**Table 2: Long-run % change in key economic variables following the imposition of the Scottish Variable Rate with endogenous population, for different values of  $\alpha$  and  $\beta$ .**

	Regional Bargaining			
	$\alpha \equiv \beta \equiv 1$	$\alpha \equiv 0 \beta \equiv 1$	$\beta \equiv 0$	$\alpha \equiv 0.8 \beta \equiv 1.2$
GDP (Income measure)	0.47	-1.97	-2.18	0.42
Consumption	-1.23	-2.37	-2.47	-1.26
Govt expend.	3.77	1.69	1.52	3.72
Investment	0.11	-2.16	-2.35	0.06
Nominal pre-tax wage	0.00	2.39	2.60	0.05
Real post-tax wage	-1.93	-0.15	0.00	-1.89
Total employment	0.68	-1.95	-2.17	0.62
Manufacturing	-0.02	-2.61	-2.83	-0.08
Non-Manufacturing	-0.28	-3.03	-3.26	-0.35
Sheltered	2.31	-0.16	-0.36	2.26
Unemployment Rate	0.00	1.35	0.00	0.32
Total population	0.68	-1.71	-2.17	0.67
Price of value added				
Manufacturing	0.00	1.76	1.92	0.04
Non-Manufacturing	0.00	1.56	1.69	0.04
Sheltered	0.00	1.94	2.11	0.04
Shadow price of capital				
Manufacturing	0.00	0.48	0.52	0.01
Non-Manufacturing	0.00	0.48	0.52	0.01
Sheltered	0.00	0.48	0.52	0.01
Consumer Price Index	0.00	0.00	0.00	0.00
	0.00	0.57	0.62	0.01
Value added				
Manufacturing	-0.02	-2.43	-2.63	-0.08
Non-Manufacturing	-0.26	-2.77	-2.97	-0.32
Sheltered	2.26	-0.10	-0.30	2.20
Capital stocks				
Manufacturing	-0.02	-2.06	-2.23	-0.07
Non-Manufacturing	-0.24	-2.44	-2.63	-0.29
Sheltered	2.01	0.13	-0.03	1.97
Exports				
Manufacturing	0.00	-1.73	-1.88	-0.04
Non-Manufacturing	0.00	-1.80	-1.95	-0.04
Sheltered	0.00	-2.70	-2.92	-0.06

**Table 3: Long-run % change in key economic variables following the imposition of the Scottish Variable Rate with population fixed**

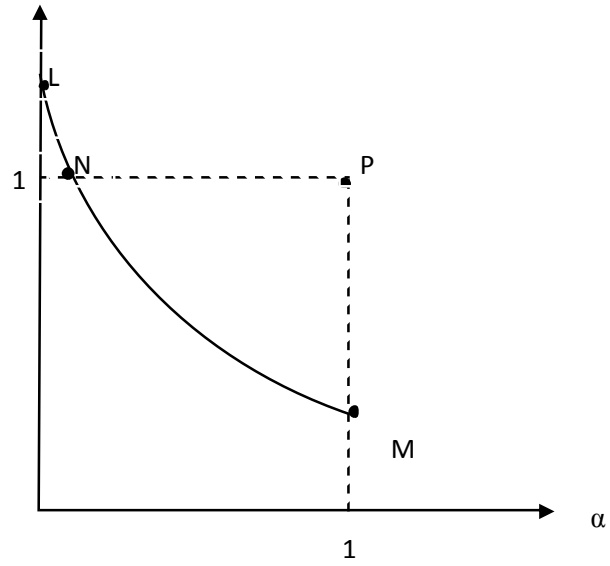
Fixed Population	Regional Bargaining		
	$\alpha \equiv \beta \equiv 1$	$\alpha \equiv 0, \beta \equiv 1$ $\beta \equiv 0$	$\alpha \equiv 0.8, \beta \equiv 1.2$
GDP (Income measure)	0.17	-1.22	0.11
Consumption	-1.38	-2.02	-1.40
Govt expend.	3.51	2.33	3.46
Investment	-0.18	-1.46	-0.23
Nominal pre-tax wage	0.30	1.65	0.35
Real post-tax wage	-1.71	-0.70	-1.67
Total employment	0.35	-1.14	0.29
Manufacturing	-0.35	-1.81	-0.41
Non-Manufacturing	-0.63	-2.19	-0.69
Sheltered	2.00	0.60	1.95
Unemployment Rate	-1.97	6.46	-1.63
Total population	0.00	0.00	0.00
Price of value added			
Manufacturing	0.22	1.21	0.26
Non-Manufacturing	0.19	1.07	0.23
Sheltered	0.24	1.34	0.28
Shadow price of capital			
Manufacturing	0.06	0.33	0.07
Non-Manufacturing	0.06	0.33	0.07
Sheltered	0.06	0.33	0.07
Consumer Price Index	0.07	0.39	0.08
Value added			
Manufacturing	-0.32	-1.69	-0.38
Non-Manufacturing	-0.58	-2.00	-0.64
Sheltered	1.96	0.63	1.91
Capital stocks			
Manufacturing	-0.28	-1.43	-0.32
Non-Manufacturing	-0.52	-1.77	-0.57
Sheltered	1.78	0.71	1.73
Exports			
Manufacturing	-0.22	-1.20	-0.26
Non-Manufacturing	-0.23	-1.25	-0.27
Sheltered	-0.34	-1.87	-0.40

**Table 4. Long-run % change in employment after the introduction of the Scottish Variable Rate with population fixed.**

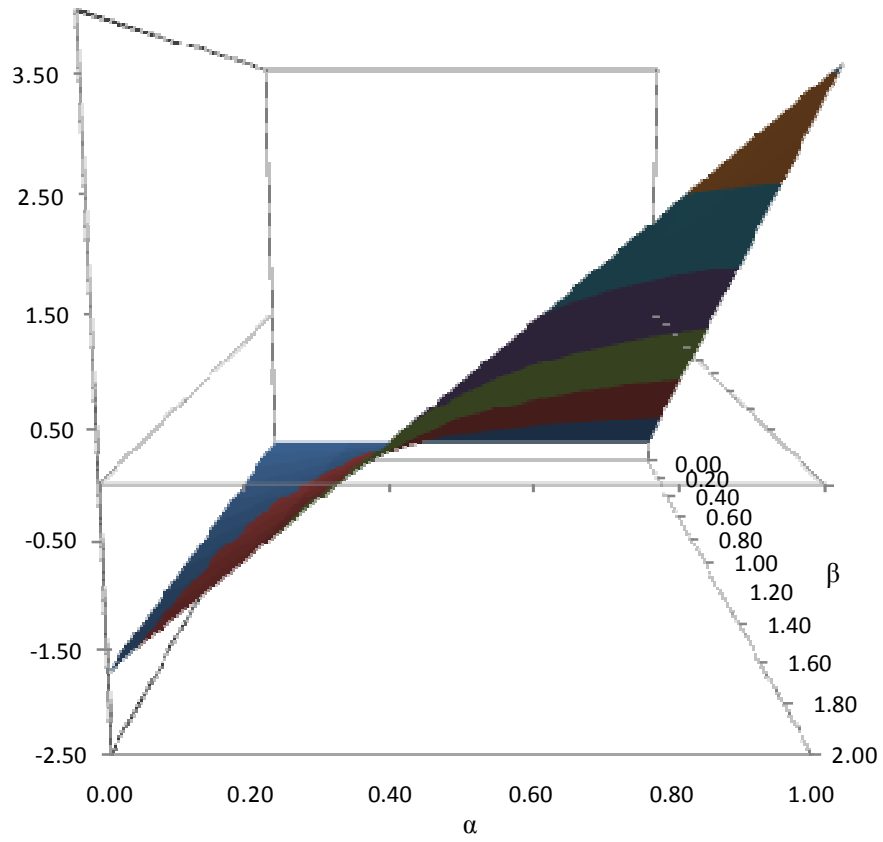
		$\alpha$					
		0.00	0.20	0.40	0.60	0.80	1.00
$\beta$	2.00	-1.14	-0.54	0.05	0.64	1.21	1.78
	1.80	-1.14	-0.60	-0.06	0.46	0.99	1.50
	1.60	-1.14	-0.66	-0.18	0.29	0.75	1.21
	1.40	-1.14	-0.72	-0.30	0.11	0.52	0.93
	1.20	-1.14	-0.78	-0.42	-0.06	0.29	0.64
	1.00	-1.14	-0.84	-0.54	-0.24	0.05	0.35
	0.80	-1.14	-0.90	-0.66	-0.42	-0.18	0.05
	0.60	-1.14	-0.96	-0.78	-0.60	-0.42	-0.24
	0.40	-1.14	-1.02	-0.90	-0.78	-0.66	-0.54
	0.20	-1.14	-1.08	-1.02	-0.96	-0.90	-0.84
	0.00	-1.14	-1.14	-1.14	-1.14	-1.14	-1.14



Figure 2. The zero employment change locus



**Figure 3. The long run change in employment after the introduction of the Scottish Variable Rate.**



**Figure 5. Percentage change adjustments in Tobin's q over time**

*Figure 5a:  $\alpha = \beta = 1$*

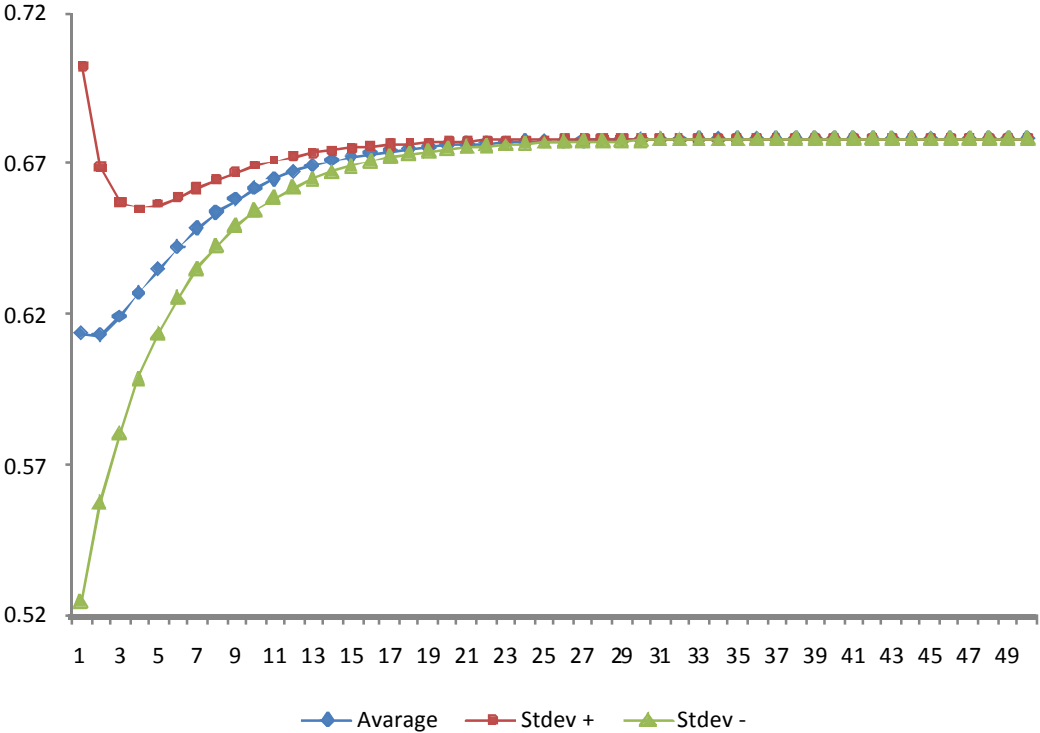


*Figure 5b:  $\alpha = \beta = 0$*





**Figure 6. The sensitivity of the percentage change in total employment to parameter variability (for  $\alpha = \beta = 1$ )**



**Figure 7. The sensitivity of the percentage change in total employment to parameter variability (for  $\alpha = 0.8$  and  $\beta = 1$ )**

