Measuring Regional Inequality:
To Weight or not to Weight?

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ABSTRACT *Estimating regional inequality, many economists use inequality indices weighted by regions' proportions of the national population. Despite this approach is widespread, its adequacy has not received attention in the regional science literature. This paper proves that such approach is conceptually inconsistent, yielding an estimate of interpersonal inequality among the whole population of the country rather than an estimate of regional inequality. But, as a measure of interpersonal inequality, such an estimate is very rough (up to misleading) and not always has an intuitive interpretation. Moreover, the population-weighted inequality indices do not meet requirements to an adequate inequality measure.*

KEYWORDS: *inequality index; weighting by population; Williamson coefficient of variation; inequality axioms*

JEL CLASSIFICATION: D31; D63; R10

1. Introduction

Studying economic inequality in a country, one may consider distribution of income between individuals or between country's regions. The latter not only introduces spatial dimension in studies of inequality, but can also reveal important links remaining overlooked with treating the country as a whole. For example, while the literature on civil war has found little support for a link between individual-level economic inequality and civil war, Deiwiks et al. (2012) find strong evidence that regional inequality affects the risk of secessionist conflict. In both cases, the same statistical methodology and inequality indices (which amount to a few tens) are applied, with the difference that regions rather than individuals are taken as observations. However, there is a modification of the inequality indices that is applied to measure regional inequality.

Apparently, Williamson (1965) was the first who put forward the idea of weighting indices that measure inequality between regions of a country by regions' shares in the national population. Since then such an approach became fairly widespread in regional studies. Publications that use it number in hundreds. Therefore I am able to cite only a small part of them, using a dozen of recent journal articles as a 'sample'. Table 1 tabulates them, reporting population-weighted inequality indices applied as well as geographical and temporal coverage of respective studies. In this table, CV = coefficient of variation, G = Gini index, Th = Theil index, MLD = mean logarithmic deviation, σ = standard deviation of logarithms and RMD = relative mean deviation. Subscript w indicates that the index is a population-weighted one.

Most studies from Table 1 use regional GDP per capita as a well-being indicator. An exception is Doran & Jordan (2013) who exploit regional gross value added per capita; a few studies consider some additional indicators. The table shows that the application of the population-weighted inequality indices is greatly varied both in geographical terms and time spans (note that if different countries are involved in a study, the case at hand is not international inequality; the study deals with regional inequalities in a respective set of countries). Inequality indices employed are also manifold. The most popular ones are the coefficient of variation, Gini and Theil indices (many other, 'out of sample', papers confirm this). Therefore only these three indices will be dealt with in what follows. It should be noted that the population-weighted indices are present not only in the literature on economic inequality; they find use in studies of inequality in the areas of health care, education, energy policy, etc.

Table 1. Selected recent studies that use population-weighted inequality indices.

Author(s)	Weighted index(es) employed	Geographical coverage	Time span
Doran & Jordan (2013)	Th_w	14 EU countries (NUTS 2 regions)	1980–2009
Enflo & Rosés (2015)	MLD_w	Sweden	1860–2000
Ezcurra & Rodríguez-Pose (2014)	Th_w, CV_w, MLD_w, σ_w	22 emerging countries	1990–2006
Kyriacou & Roca-Sagalés (2014)	CV_w, MLD_w, σ_w	22 OECD countries	1990–2005
Lessmann (2014)	CV_w	56 countries	1980–2009
Li & Gibson (2013)	G_w, CV_w, Th_w	China	1990–2010
Martínez-Galarraga et al. (2015)	Th_w	Spain	1860–2000
Mussini (2015)	G_w	28 EU countries (NUTS 3 regions)	2003–2011
Petrakos & Psycharis (2016)	CV_w	Greece	2000–2012
Sacchi & Salotti (2014)	CV_w, σ_w, MLD_w	21 OECD countries	1981–2005
Wijerathna et al. (2014)	G_w, CV_w, RMD_w	Sri Lanka	1996–2011
Zubarevich & Safronov (2011)	G_w, CV_w	Russia, Ukraine, Kazakhstan	1998–2009

Williamson did not provide a more or less detailed substantiation of his idea, merely noted that an unweighted inequality index "will be determined in part by the somewhat arbitrary political definition of regional units" and "[t]he preference for an unweighted index over a weighted one, we think, is indefensible" (Williamson, 1965, pp. 11, 34). Nor such substantiations appeared within next 50 years. Even a handbook chapter on measuring regional divides only asserts that the use of unweighted inequality indices "may lead to unrealistic results in certain cases, affecting our perception of convergence or divergence trends" (Ezcurra & Rodríguez-Pose, 2009, p. 332), providing no proof or example. A sole attempt to explore properties of the population-weighted indices is due to Portnov & Felsenstein (2010); it will be discussed in Section 5. Yet even Williamson's brief notes cited are open to question.

First, the political division of a country is the reality which regional researchers should deal with, irrespective of whether they believe it to be 'somewhat arbitrary' or 'natural'. Certainly, they may discuss its shortcomings and find ways of improvement, but it is a quite different story unrelated to the issue of regional inequality. Therefore the desire for 'adjustment' of existing political division through assigning less importance to lesser populated regions seems strange.

Second, why do we need taking into account differences in regional population at all? But we can estimate inequality among groups in country's population without regard for sizes of these groups. For instance, while estimating wage inequality between industrial workers, builders, teachers, lawyers and so on, we do not care about what shares of these occupational groups in the total population (or employees) are. What is a fundamental difference between this and the case when each population group consists of inhabitants of one region?

Third, on closer inspection results of estimating inequality with the use of population-weighted indices look striking; they may prove to be evidently unrealistic. The next section gives an impressive example.

The purpose of this paper is to show that the application of population-weighted indices for measuring regional inequality is nothing but a fallacy. The main point is that they measure not inequality between regions but something else and therefore yield distorted estimates of regional inequality. In other words, the unweghted and weighted indices **measure different phenomena**. Albeit Williamson's approach has received some criticism (which will be discussed in Section 5), the literature has overlooked this point. Moreover, this paper proves that these indices do not meet requirements to an adequate inequality measure.

The statement above seemingly contradicts the fact that the approach under consideration is commonly employed in the literature. However, this fact in no way evidences adequacy of the approach. For instance, analyzing β-convergence is even more widespread (in the literature on economic growth and inequality); relevant publications number in thousands. Nonetheless, a number of authors, e.g. Friedman (1992), Quah (1993), Wodon & Yitzhaki (2006) and Gluschenko (2012), proved invalidity of this methodology.

The rest of the paper is organized as follows. Section 2 reveals the true sense of inequality estimates obtained with the use of the population-weighted indices. Section 3 considers the issues of biases in the weighted indices and interpretability of these indices. Section 4 analyzes properties of the population-weighted indices, providing proofs that they violate three important axioms. Section 5 discusses arguments against and in favour of the population weighting found in the literature. Section 6 summarizes conclusions drawn in the paper.

2. What Do Population-Weighted Indices Measure?

Consider cross-region income distribution $y = (y_i)$, $i = 1, \ldots, m$; y_i = per capita income in region i and $\bar{y}$ = the arithmetic average of regional per capita incomes ($\bar{y} = (y_1 + ... + y_m)/m$). Then the coefficient of variation measuring regional inequality has the form

$$CV = \frac{\sqrt{\sum_{i=1}^{m}(y_i - \bar{y})^2 / m}}{\bar{y}}. \tag{1}$$

Now let N_i = population of region i; N = population of the country; $n_i = N_i/N$ = region's share in the national population (region's weight); $n = (n_i)$ will be called population distribution. The weighted average of regional per capita incomes ($\bar{y}_{(w)} = n_1 y_1 + ... + n_m y_m$) is denoted by $\bar{y}_{(w)}$. It exactly equals the national per capita income: $\bar{y}_{(w)} = (Y_1 + ... + Y_m)/N = Y/N$, where Y_i stands for region's total income ($Y_i = N_i y_i$) and Y represents the national total income. Under this notation, the Williamson coefficient of variation (Williamson, 1965, p. 11) – sometimes called the Williamson index – looks like

$$CV_w = \frac{\sqrt{\sum_{i=1}^{m}(y_i - \bar{y}_{(w)})^2 n_i}}{\bar{y}_{(w)}}. \tag{2}$$

The Gini and Theil indices can be respectively written as

$$G = \frac{\sum_{i=1}^{m}\sum_{k=1}^{m}|y_i - y_k|}{2m^2\bar{y}}; \tag{3}$$

$$Th = \frac{1}{m}\sum_{i=1}^{m}\frac{y_i}{\bar{y}}\ln(\frac{y_i}{\bar{y}}). \tag{4}$$

Their population-weighted counterparts take the forms

$$G_w = \frac{\sum_{i=1}^{m}\sum_{k=1}^{m}n_i n_k|y_i - y_k|}{2\bar{y}_{(w)}}; \tag{5}$$

$$Th_w = \sum_{i=1}^{m}n_i\frac{y_i}{\bar{y}_{(w)}}\ln(\frac{y_i}{\bar{y}_{(w)}}). \tag{6}$$

Sometimes, the weighting by population is present in the Theil index implicitly. For example, Doran & Jordan (2013, p. 25–26) construct the index from regions' contributions to the total income, Y_i/Y, and regions' contributions to the total population, N_i/N. Martínez-Galarraga et al. (2015, p. 510) use a similar way. It is easily seen that such index is equivalent to that represented by Formula (6):

$$\sum_{i=1}^{m}\frac{Y_i}{Y}\ln(\frac{Y_i/Y}{N_i/N}) = \sum_{i=1}^{m}\frac{N_i y_i}{N\bar{y}_{(w)}}\ln(\frac{Y_i/N_i}{Y/N}) = \sum_{i=1}^{m}n_i\frac{y_i}{\bar{y}_{(w)}}\ln(\frac{y_i}{\bar{y}_{(w)}}) = Th_w.$$

Let us perform a simple test for adequacy of the population-weighted indices. Consider two Chinese regions, mainland China as a whole (in Chinese, Dàlù) and Macao, the Special Administrative Region of the People's Republic of China (and the richest territory of the world). In hoary antiquity, when the Portuguese occupied as large part of the Chinese territory as they could (or needed), Macao might be deemed a 'somewhat arbitrary' regional unit.

Nowadays, it is quite natural, as Macao has its own currency, and citizens of China from other regions need visa to get there. Table 2 reports data on these regions.

Table 2. Per capita income and population in mainland China and Macao in 2014.

Region	PPP-adjusted GDP per capita (y_i), current international dollars [1]	Population (N_i), million people [2]	Region weight (n_i)
Mainland China	13,217	1,376.049	0.999573
Macao	139,767	0.588	0.000427

[1] World Bank (2015).

[2] United Nations (2015, p. 13).

Estimating income inequality between mainland China and Macao, we get results listed in Table 3. It reports values of the population-weighted coefficient of variation and Gini and Theil indices defined by Formulae (2), (5) and (6) as well as values of the unweighted indices according to Formulae (1), (3) and (4). For comparability, the table also reports these indices standardized so that they range from 0 to 1. That is, an index is divided by its maximum corresponding to perfect inequality. For our case of two observations, the maxima of CV, G and Th are respectively 1, 0.5 and log(2). The maxima of CV_w, G_w and Th_w approximately equal 1, 48.4 and 7.8 (the way of computing these maxima will be explained in Section 4 and summarized in its Table 8).

Table 3. Estimates of income inequality between mainland China and Macao.

Index	Population-weighted		Unweighted	
	Raw	Standardized	Raw	Standardized
Coefficient of variation	0.197	0.004	0.827	0.827
Gini index	0.004	0.004	0.414	0.827
Theil index	0.007	0.001	0.399	0.576
Average income	$\bar{y}_{(w)}$ = 13,721		$\bar{y}$ = 76,492	

While the unweighted indices indicate a high degree of inequality, the population-weighted ones yield the reverse pattern. The standardized values of CV_w and G_w are equal to 0.4% in percentage terms; and standardized Th_w is even less than 0.1%. This suggests that there is (almost) no income inequality between the average mainland Chinese and average inhabitant of Macao. Indeed, our perception of spatial inequality is greatly distorted, but in the sense opposite to the above-cited view of Ezcurra & Rodríguez-Pose (2009, p. 332): it is the population weighting that gives rise to distortions.

In the two-region case, the result evidently contradicts common sense. A sufficiently great number of regions in empirical studies masks, as a rule, such absurdities, creating an impression that estimates of inequality with the use of the population weighting are reasonable.

Then what is the reason for that low inequality suggested by the population-weighted inequality indices in the above example? What is the sense of the estimates obtained? To understand what the weighted indices measure, let us estimate inequality among all citizens of a country, basing on cross-region income distribution. The 'national' coefficient of variation (CV_{nat}) with y_l standing for personal income of l-th citizen of the country looks like

$$CV_{nat} = \frac{\sqrt{\sum_{l=1}^{N}(y_l - \bar{y})^2 / N}}{\bar{y}}.$$

Obviously, the population-average income in this formula – national per capita income, $\bar{y} = (y_1 + ... + y_N)/N$ – equals the weighted average of regional per capita incomes, $\bar{y}_{(w)}$. Lacking information on intra-regional income distributions, we are forced to assume that all inhabitants of a region have the same income equalling per capita income in this region. Then square deviations $(y_l - \bar{y}_{(w)})^2$ are uniform for all l relating to inhabitants of the same region, say i. Hence, their sum over all inhabitants of the region is $(y_i - \bar{y}_{(w)})^2 N_i$. Summing up such sums over all regions, we come to the Williamson coefficient of variation:

$$CV_{nat} = \frac{\sqrt{\sum_{i=1}^{m}(y_i - \bar{y}_{(w)})^2 N_i / N}}{\bar{y}_{(w)}} = \frac{\sqrt{\sum_{i=1}^{m}(y_i - \bar{y}_{(w)})^2 n_i}}{\bar{y}_{(w)}} = CV_w.$$

Thus, the population-weighted coefficient of variation **is not a measure of inequality between regions**; instead, **it measures national inequality**, i.e. interpersonal inequality in the whole population of the country. This fact is valid for any other population-weighted inequality index (maybe, except for those based on partial information from cross-region income distributions, e.g. the relative range of disparities, $R = \max_i y_i / \min_i y_i$, interquartile range, and the like; however, it seems that the weighting is hardly applicable to them); the proof is simple and similar to the above one. (Of course, we need not necessarily take an entire country; the conclusion still holds if we consider any subset of regions as a 'country'.)

This explains the sense of results obtained with the population-weighted indices in Table 3. These measure inequality between inhabitants of united mainland China and Macao. Provided that inequality within mainland China is zero (as all its inhabitants are supposed to have the same income), adding less than one million people – even with extremely high income – to its 1.4-billion population can increase the degree of the overall inequality only slightly.

It is seen that there is a conceptual distinction between the unweighted and population-weighted estimates of inequality; **they measure different phenomena**. The unweighted index measures inequality between regions (considered as a whole), while the weighted one measures inequality between all country's citizens.

Considering inequality between regions, all of them enjoy equal rights in the sense that all y_i are equiprobable (i.e. the probability of finding income y_i in a randomly chosen region is the same for all i and equals $1/m$). Albeit speaking of regions, we actually deal with individuals, representative (or 'average', i.e. having the region-average income) inhabitant of each region. Estimating regional inequality, we compare their incomes without regard for how many people live in respective regions (like we do while comparing wages across occupations). Indeed, the fact that the average inhabitant of Macao is almost 11 times richer than the average mainland Chinese in no way changes because of the fact that the population of Macao is 2,340 times smaller than the population of mainland China.

Introducing regional weights implies that a region is represented by all its inhabitants rather than by one 'average' inhabitant. That is, we consider region i as a group of N_i people, each individual within the group having income y_i. Then the probability of y_i differs across regions, becoming proportional to their populations, n_i. Thus, (n_i) is a proxy of the personal-income distribution in the country. In fact, we 'split' regions into their individual inhabitants so that all they in aggregate represent the whole population of the country, as Figure 1 illustrates, and estimate inequality between these N persons, so inevitably substituting regional inequality for interpersonal one. However, lacking information on income differences within regions, we consider inhabitants of each region as identical and regions as internally homogeneous groups of people. Thus we arrive at a grouping of the whole country's population into income classes (y_i) of different sizes (N_i). The regional division matters no more; the impression that the case at hand is inequality between regions is but an illusion owing to that the grouping proceeds from the data by region. An estimate of the national inequality obtained with such grouping is very crude, since it neglects inequality within regions and – what is much more important – the income classes y_i (constructed from cross-region data) in fact heavily overlap because of overlapping intra-regional income distributions. (This issue is considered in more detail in the next section.)

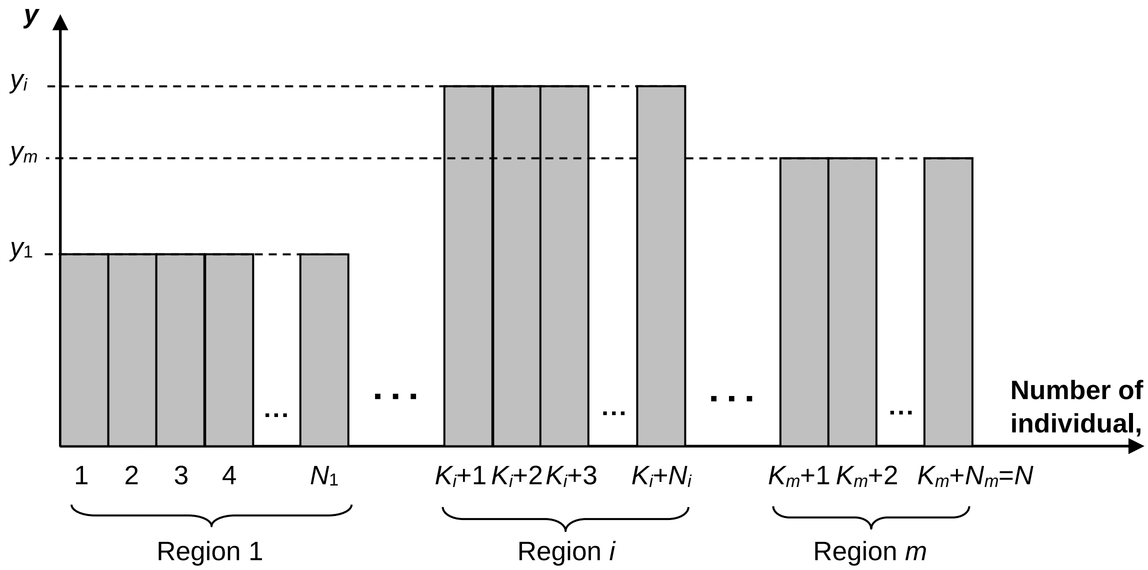


Figure 1. Population of the country as a set of regional populations (assuming population of each region to be income homogeneous, $y_l = y_i$ for $K_i + 1 \le l \le K_i + N_i$).

Note: $K_i = \sum_{j=1}^{i-1} N_j$; $K_1 = 0$.

As it is known, inequality in the country, being measured by the Theil index, can be decomposed into two components: within-region inequality and between-region inequality. Under notation of this paper, the decomposition of national inequality looks like

$$Th_{nat} = \sum_{i=1}^{m} n_i \frac{y_i}{\bar{y}_{(w)}} Th_i + \sum_{i=1}^{m} n_i \frac{y_i}{\bar{y}_{(w)}} \log(\frac{y_i}{\bar{y}_{(w)}}) = \sum_{i=1}^{m} n_i \frac{y_i}{\bar{y}_{(w)}} Th_i + Th_w ,$$

where Th_i = Theil index for the population of *i*-th region. Thus, the population-weighted Theil index represents only a part of national inequality, namely, between-region inequality. It answers to the counterfactual question: 'how much inequality would be observed [in the country] if there was no inequality within regions?' (Shorrocks & Wan, 2005, p. 60).

It follows herefrom that a population-weighted estimate of inequality is biased with regard to estimates of both regional inequality (as it measures a different value) and interpersonal inequality (as it does not take account of within-region income disparities). In both cases, the result can be misleading as the example of two Chinese regions demonstrates.

The bias can have either direction depending on a particular combination of regional per capita incomes and populations. Williamson (1965, p. 12) reports values of both weighted and unweighted coefficient of variation estimated on regional data from 24 countries. Regional inequality estimated by CV_w proves to be overstated in about a half of countries, and understated in another half. The biases (relative to the unweighted estimates) range from

–52.6% (in India) to +37.6% (in Puerto Fico). The case of India is an example of quite misleading result in an actual study (covering 18 regions): the population-weighted index understates the extent of regional inequality there by more than a half. Differences in trends also can occur. Figure 2 depicts the evolution of inequality in Australia over 11 years according to the unweghted and population-weighted coefficient of variation. The estimates are computed from Williamson's (1965, p. 48) data. It is seen that the trends of CV and CV_w are sometimes opposite, e.g. in the whole period of 1952/53 to 1958/59. Regional inequality, measured by CV, fell by 4.7% in 1959/60 as compared to 1949/50, while it increased by 20.6% according to the weighted estimates. Thus we come to opposite conclusions depending on the use of unweghted or population-weighted measures.

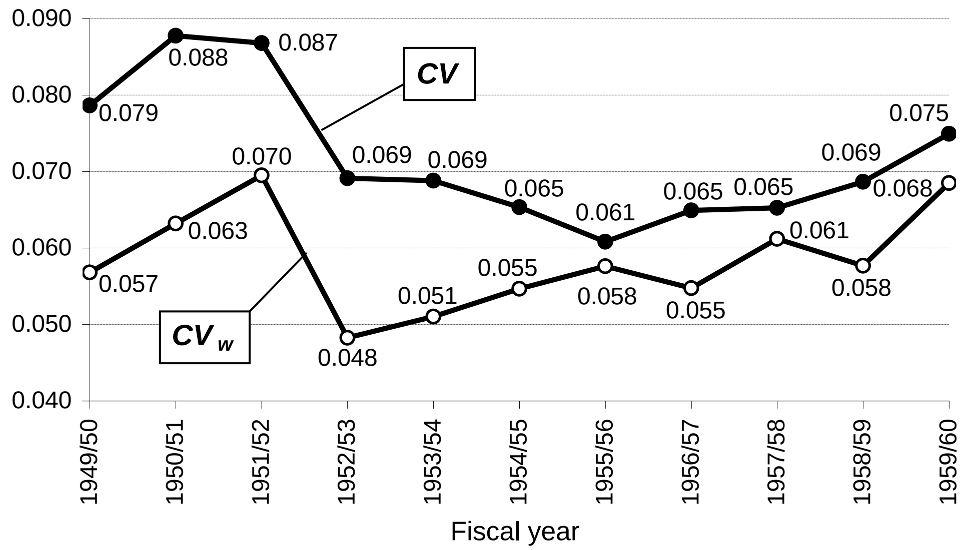


Figure 2. Paths of the weighted and unweighted coefficient of variation in Australia.

One more evidence is due to Petrakos & Psycharis (2016). They estimate the evolution of regional inequality in Greece across its NUTS 2 and NUTS 3 regions over 2000–2012, using both population-weighted and unweighted coefficient of variation. The trend of CV_w is upward, while CV has either a downward trend (in the case of NUTS 3 regions) or is stable (for NUTS 2 regions).

Milanovic's (2012) results provide evidence in the international context. He estimates income inequality (by the Gini index) between counties and in the world as a whole over 1952–2006. In the latter case, the index weighted by populations of the countries is applied. However, unlike most (if not all) regional studies applying population-weighted measures, he explicitly interpret it as an approximate measure of global inequality (inequality across world individuals) rather than an estimate of international (cross-country) inequality, realizing that it is not only a rough, but possibly misleading, estimate. The sole reason for application of that rough proxy is the absence of household survey data for a sufficient number of countries prior to the 1980s (Milanovic, 2012, p. 8). The trends of the unweighted and weighted Gini indices are found to have opposite directions, upward for the former and downward for the latter (the both become downward only since 2000). As regards interpersonal inequality, Milanovic (2012, p. 14) also reports estimates of global inequality for 1988–2005 based on household survey data (i.e. taking into account income distributions within countries). These prove to be, first, much higher that the weighted estimates, and, second, sliding upward (although only slightly) rather than downward. Thus, estimates obtained with the use of the weighted Gini index turn out to be really misleading with respect to inequality both between countries and between world individuals.

3. Population-Weighted Indices as Measures of Interpersonal Inequality

As it has been mentioned in the previous section, an estimate of interpersonal inequality in the country or any subset of regions with the use of the population-weighted indices is biased because of neglecting inequality within regions. An actual 'splitting' regions into their individual inhabitants would yield something like a pattern depicted in Figure 3 (individuals within each region are arranged according to their personal incomes), which fundamentally differs from the pattern supposed in Figure 1.

Individuals in a region may have incomes that are near to or even coincide with incomes of inhabitants of other regions, which implies that individuals from the same region in fact fall into different income classes and individuals from different regions may fall into the same income classes. In other words, regional income distributions overlap with one another. Because of this overlapping, the division of country's population into income classes according to regional per capita income – like in Figure 1 – turns out improper, resulting in inadequate estimation of inequality in the country. To correctly estimate inequality between N persons making up the population of the country, all they should be rearranged by income within the whole country and then grouped (irrespective of their regions of origin) into some actual income classes.

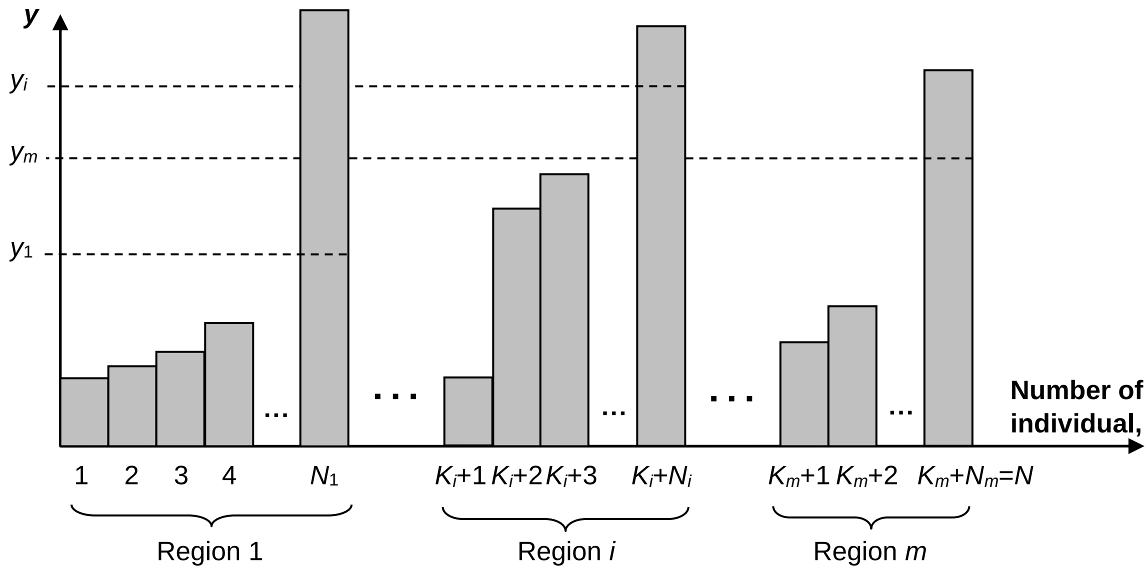


Figure 3. Population of the country as a set of regional populations (actual pattern).
Note: $K_i = \sum_{j=1}^{i-1} N_j$; $K_1 = 0$.

Applying the population-weighted Gini index, Mussini (2015) estimates inequality between NUTS 3 regions in the EU-28 over 2003–2011 and decomposes its changes into those caused by population change, re-ranking of regions and growth of regional per capita incomes. In the light of the above considerations the intuitive sense of the first component becomes absolutely obscure. In fact, inequality within population of the whole geographical entity comprising of NUTS 3 regions is measured. Imagine that the cross-individual income distribution in this entity remains invariant while the cross-region population distribution changes. Then the effect of population change in the decomposition of inequality change reflects nothing but a result of replacing one improper division of the population into income classes by another, also improper, one.

Data drawn from the Russian statistics provide convenient real examples with small numbers of regions that make it possible to judge the extent of distortions in estimates of interpersonal inequality caused by the application of population-weighted indices. There are two regions in Russia, the Arkhangelsk Oblast and Tyumen Oblast, that include national entities, so called autonomous *okrugs* (hereafter, AO). The Arkhangelsk Oblast includes the Nenets AO, and the Tyumen Oblast includes Khanty-Mansi AO and Yamalo-Nenets AO. Statistical data on personal income distribution and inequality are available for each *oblast* as a whole and all its parts (‘subregions’), namely, AO(s) and the *oblast* excluding AO(s); for brevity, the latter will be called Southern part. Based on such data, we can compare actual estimates of inequality in the whole region with those obtained with the use of population-weighted index for these two-subregion and three-subregion cases. Table 4 tabulates the relevant data.

Table 4. Income and population in the Arkhangelsk and Tyumen Oblasts in 2014.

i	Region/subregion	Personal income, Russian rubles (RUR) per month [1]			Gini index [2]	Population, thousand people, annual average [3]	Subregion weight (n_i)
		Per capita (y_i)	Median (Md_i)	Modal (Mo_i)			
0	Arkhangelsk Oblast	29,432	23,125	14,276	0.378	1,187.6	
1	Nenets AO	66,491	48,281	25,457	0.429	43.2	0.036
2	Southern part	28,033	22,354	14,213	0.368	1,144.4	0.964
0	Tyumen Oblast	38,523	27,508	14,026	0.439	3,563.8	
1	Khanty-Mansi AO	41,503	30,440	16,375	0.423	1,604.7	0.450
2	Yamalo-Nenets AO	61,252	44,517	23,515	0.429	539.8	0.151
3	Southern part	26,509	20,052	11,473	0.404	1,419.3	0.398

[1] Rosstat (2016a).
[2] Rosstat (2016b).
[3] Rosstat (2015, pp. 39–40).

The Russian statistical agency, Rosstat (formerly, Goskomstat), models income distributions in regions and the whole country as log-normal ones (Goskomstat of Russia, 1996, p. 79). The distribution parameters from Table 4 enable to restore log-normal income distributions for subregions of the regions under consideration,

$f_i(y) = \frac{1}{y\sigma_i\sqrt{2\pi}}\exp(-\frac{(\log(y)-\mu_i)^2}{2\sigma_i^2})$, where $\mu_i = \log(Md_i)$ and

$\sigma_i^2 = \log(Md_i / Mo_i) = 2\log(y_i / Md_i)$. Figure 4, (a) and (c), depicts these distributions.

Estimating inequality in the whole country or – as in our case – a multi-regional entity from per capita incomes only (like the population-weighted indices do), a within-(sub)region income distribution is in fact represented as the delta function $\delta(x)$ which is zero everywhere except at zero and $\delta(0) = \infty$ so that $\int_{-\infty}^{\infty}\delta(x)dx = 1$ (see, e.g., Kanwal, 2004).[1] Denote such distribution in (sub)region i by $g_i(\cdot)$; then $g_i(y) = \delta(y - y_i)$. These distributions are represented in Figure 4 by vertical arrows starting at y_i, a number near the arrowhead specifying the area under the function.

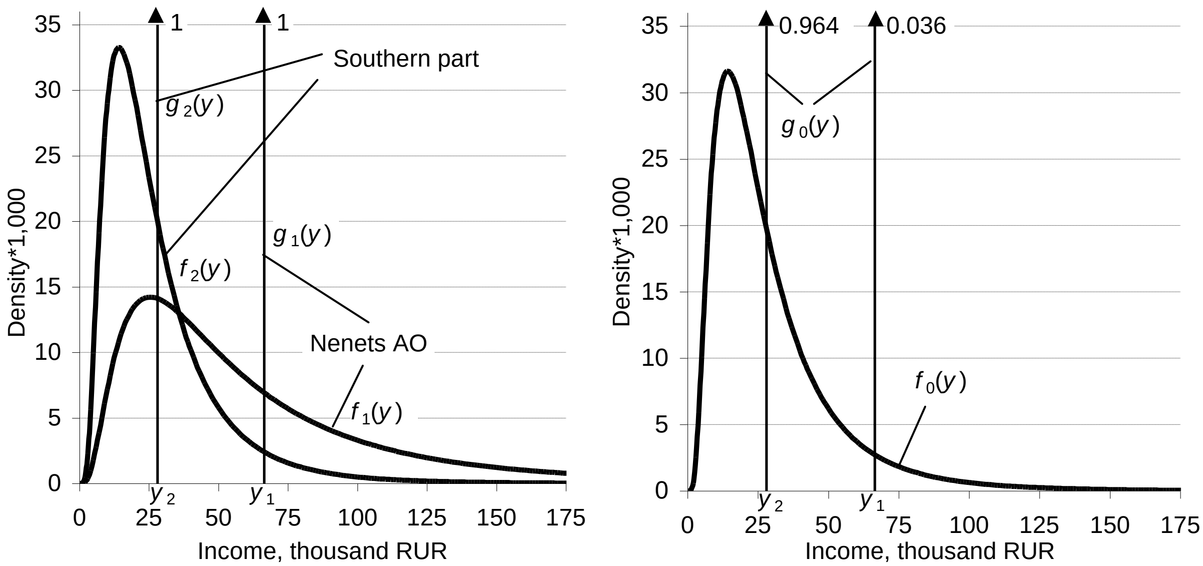
The income distribution in the whole region can be computed either in the same way as the subregional distributions (from parameters of the distribution) or equally well as the

[1] The delta function can be viewed as a limit: $\delta(x) = \lim_{\sigma\to 0}\frac{1}{\sigma\sqrt{2\pi}}\exp(-x^2 / 2\sigma^2)$.

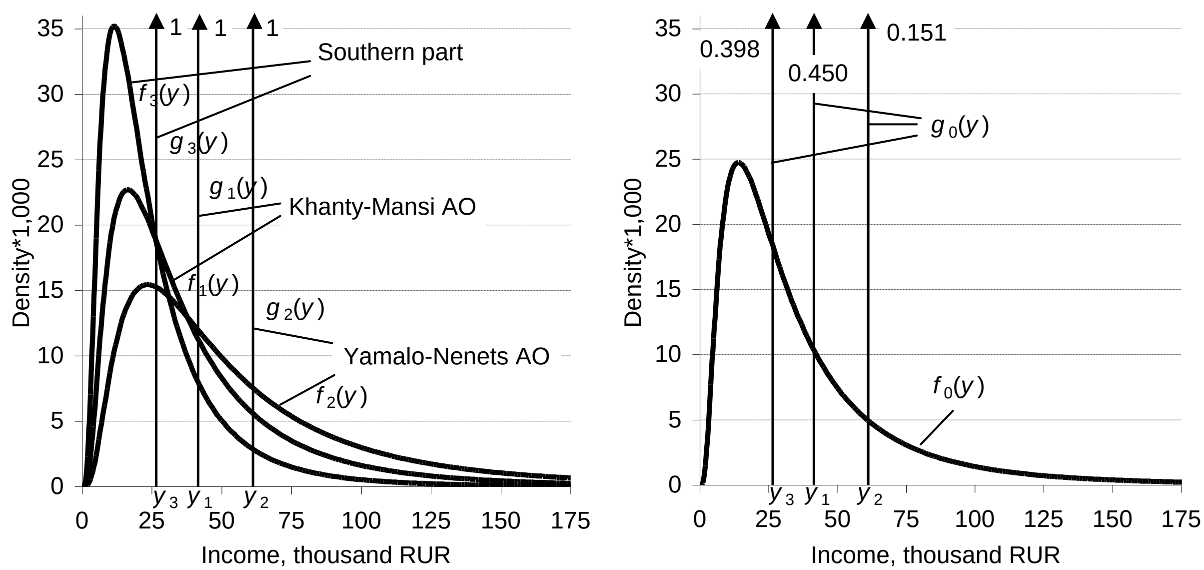
weighted sum of subregional distributions, $f_0(y) = \sum_{i=1}^{m} n_i f_i(y)$. Similarly,

$$g_0(y) = \sum_{i=1}^{m} n_i g_i(y) = \sum_{i=1}^{m} n_i \delta(y - y_i) \,. \tag{7}$$

Figure 4, (b) and (d), shows both $f_0(y)$ and $g_0(y)$.



(a) Subregions of the Arkhangelsk Oblast (b) Arkhangelsk Oblast as a whole



(c) Subregions of the Tyumen Oblast (d) Tyumen Oblast as a whole

Figure 4. Income distributions in the Arkhangelsk and Tyumen Oblasts.

Given $g_0(y)$, the expectation of y is $E_0(y) = \int_0^\infty y \sum_{i=1}^{m} n_i \delta(y - y_i) dy = \sum_{i=1}^{m} n_i y_i = \bar{y}_{(w)}$;

the variance is $Var_0(y) = \int_0^\infty (y - E_0(y))^2 \sum_{i=1}^m n_i \delta(y - y_i) dy = \sum_{i=1}^m n_i (y_i - \bar{y}_{(w)})^2$. Then the coefficient of variation coincides with that given by Formula (2), $Var_0(y)^{1/2}/E_0(y) = CV_w$. Computing the Theil index for continuous distribution, we again get its weighted version, $\int_0^\infty \frac{y}{E_0(y)} \ln(\frac{y}{E_0(y)}) g_0(y) dy = Th_w$. Expressing the Gini index as $\int_0^\infty F_0(y)(1 - F_0(y)) dy / E_0(y)$ (Yitzhaki & Schechtman, 2013, pp. 15–16 and 26), where $F_0(y)$ is the cumulative distribution function, $F_0(y) = \int g_0(y) dy$, we arrive at G_w. (The derivation needs cumbersome mathematics and therefore is not reported.) Using $f_0(y)$ instead $g_0(y)$ in the above calculations, we would get the unweighted inequality indices, *CV*, *Th* and *G*, that measure inequality of the whole population of a region for the case of continuous income distribution. (This is one more proof of the fact that the population-weighted indices measure inequality between all individuals, and not between regions.)

It is obvious – and clearly seen in Figure 4, (b) and (d), – that the approximation of the actual income distribution $f_0(y)$ by the weighted sum of delta functions, $g_0(y)$, is overly rough and therefore will never yield correct estimates of population's inequality. In Table 5, the population-weighted estimates, G_w, are compared with the estimates of inequality between subregions, *G*, and estimates of region's population inequality labelled G_{pop}. Because of small numbers of observations, *G* and G_w are standardized to range from 0 to 1 in order to make them comparable across regions. For the Arkhangelsk Oblast, the normalizing factor equals 2 for *G* and $1/(1 - n_1) = 1/0.964$ for G_w; for the Tyumen Oblast, it is equal to 3/2 and $1/(1 - n_2) = 1/0.849$, respectively (for explanation, see Table 8 in the next section).

Table 5. Estimates of inequality in the Arkhangelsk and Tyumen Oblasts.

Region	Measure	Gini index, raw / standardized	Average income, RUR
Arkhangelsk Oblast	Inter-subregional inequality (*G*)	0.203 / 0.407	$\bar{y} =$ 47,262
	Population-weighted estimate (G_w)	0.046 / 0.048	$\bar{y}_{(w)} =$ 29,432
	Population's inequality (G_{pop}) [1]	0.378	
Tyumen Oblast	Inter-subregional inequality (*G*)	0.179 / 0.269	$\bar{y} =$ 43,088
	Population-weighted estimate (G_w)	0.159 / 0.188	$\bar{y}_{(w)} =$ 38,523
	Population's inequality (G_{pop}) [1]	0.439	

[1] The official estimate from Table 4.

The case of the Arkhangelsk Oblast resembles the example of China in the previous section. Like in that example, there are two territorial units, one with big population and relatively small income per capita, and one with small population (3.6% of the total) and high income per capita (about 2.4 times higher than in the first unit). Although the difference between these subregions is not that dramatic as between mainland China and Macao, the estimation results are qualitatively similar. The weighted Gini index suggests low inequality, 4.8% in percentage terms, while inequality between the Nenets AO and Southern part – measured by the unweighted index – is rather high, 40.7%. The latter reflects the fact that the average inhabitant of the Nenets AO is 2.4 times richer than the average inhabitant of the Southern part. As for population's inequality, it equals 37.8%, only one percent point higher than inequality in the Southern part (see Table 4). A minor contribution of the Nenets AO to inequality in the whole *oblast* is just due to its small population. As it is seen in Figure 4, the overall income distribution in the whole Arkhangelsk Oblast, Figure 4 (b), differs from that in its southern part in Figure 4 (a) only slightly. The weighted Gini index – equaling 4.8% – fails to provide more or less adequate approximation of population's inequality in the whole *oblast* as well. The weighted index severely understates inequality between both subregions and inhabitants of the whole Arkhangelsk Oblast.

The case of the Tyumen Oblast involves three territorial units. They are closer to one another, both in incomes per capita and weights, than in the previous case. Smaller difference in incomes per capita results in smaller inequality between subregions (measured by G). The weighted index is again understated as compared to inequality between subregions and inhabitants of the whole Tyumen Oblast.

The patterns provided by G and G_{pop} and the differences between them in the Arkhangelsk and Tyumen Oblasts can be easily explained. High inequality between the subregions of the Arkhangelsk Oblast is due to great difference in income per capita. Interpersonal inequality is smaller than regional one, remaining approximately close to inequality in the southern part of the *oblast*, since adding rich (on average) but small population of the Nenets AO only slightly changes the income distribution. In the Tyumen Oblast, inequality between the subregions is lower because of lesser differences in incomes per capita. At the same time, inequality of the whole population of the *oblast*, G_{pop}, is higher than regional one, G, and rises as compared to inequality in each subregion. The reason is the merging of poor (on average) population of the Southern part with richer (on average) population of AOs, population sizes of the subregions being comparable. As regards results suggested by G_w, they hardly can be intuitively explained.

It is worth noting that even the interpretation of a population-weighted inequality index as an approximate measure of interpersonal inequality of the whole country's population is not always true. It holds only for indicators which can be assigned to an individual, e.g. personal income, wage, housing, education, etc. Otherwise the meaning of the population-weighted index is obscure. Estimating regional income inequality, many authors use regional GDP per capita to characterize incomes in regions. However, there is no inequality in the national GDP (as the total of regional GDPs) per capita between country's citizens. There are many other indicators that characterize situation of a region, but cannot be assigned to its certain inhabitant, e.g. birth rate, investment per capita, crime rate, etc. In such cases, the population-weighted inequality indices have no intuitive interpretation at all; it is totally incomprehensible what they measure.

For example, Zubarevich & Safronov (2011) estimate, in addition to income inequality, regional inequalities in investment per capita, unemployment rate and poverty rate. Again, there is no, e.g. unemployment inequality between country's inhabitants; only the national average unemployment rate exists. Consider a simple example. A country consists of two regions. The labouring population is made up of 15 million people in the first region and 5 million in the second; unemployment rates are 40% and 20%, respectively. Then the unemployment rate in the country is 35%. With some trick, we can measure 'unemployment inequality' of the total labouring population. A person can be either employed, y_i =1, or unemployed, y_i =0. Measuring unemployment inequality of the given 20 million persons with the Gini index over so quantified $\{y_i\}$, we get G_{nat} = 0.35, exactly the country-average unemployment rate. It can be easily checked that this is not a coincidence. Provided that the variable is binary, the Gini index always gives the percentage of zeros. At the same time, the population-weighted estimate yields G_w = 0.107; being standardized, it equals 0.107/(1 – 0.25) = 0.143. Both figures are far from the overall inequality G_{nat} as well as from inequality between regions, G = 0.167 (its the standardized value equalling 0.167/0.5 = 0.333.)

4. Some Properties of the Population-Weighted Indices

An adequate inequality index should satisfy a number of axioms, i.e., desirable properties of an inequality measure (see, e.g. Cowell, 2000). Ezcurra & Rodríguez-Pose (2009, pp. 332–333) argue – with no proof – that a number of the population-weighted inequality indices, including the coefficient of variation and Gini and Theil indices, fulfil the basic axioms, namely, scale invariance, population principle, anonymity principle and principle of transfers (the Pigou-Dalton principle). Similar assertions can be found in Kyriacou & Roca-Sagalés (2014, p. 188–189), Lessmann (2014, p. 37), Sacchi & Salotti (2014, p. 148–149) and elsewhere.

Indeed, these indices are scale-invariant; the check is easy and straightforward. As regards the population principle, anonymity (symmetry) principle and principle of transfers, the population-weighted inequality indices violate them (while their unweighted counterparts do satisfy).

The population principle (or replication invariance) states that a simple replication of the sample under consideration should not change the value of the inequality index. Let us replicate the income distribution (y_i), along with the population distribution (n_i), R times, indicating new values of variables by superscript (R). The population-weighted coefficient of variation takes the form $CV_w^{(R)} = \sqrt{(CV_w^2+1)/R+R-2}$; it increases with rising R. The weighted Gini index becomes R times greater: $G_w^{(R)} = RG_w$. The weighted Theil index, contrastingly, diminishes: $Th_w^{(R)} = Th_w - \log(R)$, taking on negative values. (Note that the weighted average also changes because of replication: $\bar{y}_{(w)}^{(R)} = R\bar{y}_{(w)}$).

The violation of the anonymity and transfer principles will be proved below for the population-weighted coefficient of variation. Such proofs for the population-weighted Gini and Theil indices need more cumbersome mathematics; therefore only numerical examples will illustrate violations of these axioms by them.

Adjusting Jenkins & van Kerm's (2009, p. 52) definition to the case of regions, the anonymity principle requires the inequality index to depend only on per capita income values used to construct it and not additional information such as what the region is with a particular per capita income or what regional populations are. In other words, the index must be invariant to any permutation of income observations.

Consider a cross-region income distribution $y = (y_1,\ldots, y_N)$ and its permutation y_*, i.e., $y = (\ldots, y_i, \ldots, y_k, \ldots)$ and $y_* = (\ldots, y_k, \ldots, y_i, \ldots)$; the rest elements in y_* remain the same as in y; hereafter $y_k > y_i$. One can expect the value of the population-weighted inequality index to change under such a transformation if for no other reason than it changes the weighted average:

$$\Delta\bar{y}_{(w)} = \bar{y}_{*(w)} - \bar{y}_{(w)} = (n_i - n_k)(y_k - y_i)\,. \tag{8}$$

It is seen that the weighted average remains intact only in the trivial case of $n_i = n_k$.

The change in the population-weighted coefficient of variation is characterized by the following equation:

$$\Delta CV_w^2 = CV_w^2(y_*) - CV_w^2(y) = \frac{\Delta\bar{y}_{(w)}}{(\bar{y}_{(w)} + \Delta\bar{y}_{(w)})^2}(y_i + y_k - (2\bar{y}_{(w)} + \Delta\bar{y}_{(w)})\frac{\overline{y^2}_{(w)}}{\bar{y}^2_{(w)}}), \tag{9}$$

where $\overline{y^2}_{(w)}$ is the weighted average of squared incomes and $\bar{y}^2_{(w)}$ is the square of the weighted average; $\Delta\bar{y}_{(w)}$ is defined by Formula (8). Note that $\overline{y^2}_{(w)} / \bar{y}^2_{(w)} = CV_w^2(y) + 1$; hence, it always exceeds unity. Thus, ΔCV_w^2 depends on six variables: y_i, y_k, n_i, n_k, $\bar{y}_{(w)}$, and $\overline{y^2}_{(w)}$. (This number may be reduced by one, replacing the latter two variables with $CV_w(y)$.) The signs of the relationship

$$\frac{y_i + y_k}{2\bar{y}_{(w)} + \Delta\bar{y}_{(w)}} \cdot \frac{\bar{y}^2_{(w)}}{\overline{y^2}_{(w)}} - 1 \equiv H(y_i, y_k, n_i, n_k, \bar{y}_{(w)}, \overline{y^2}_{(w)}) - 1$$

and $\Delta\bar{y}_{(w)}$ determine the sign of ΔCV_w^2, hence the direction of change in the inequality measure: $\operatorname{sgn}(\Delta CV_w^2) = \operatorname{sgn}(H(\cdot) - 1) \cdot \operatorname{sgn}(\Delta\bar{y}_{(w)})$. Table 6 shows different possible cases.

Table 6. Permutation-induced changes in the population-weighted coefficient of variation.

	$n_i > n_k$ ($\Delta\bar{y}_{(w)} > 0$)	$n_i < n_k$ ($\Delta\bar{y}_{(w)} < 0$)
$H(\cdot) > 1$	CV_w increases	CV_w decreases
$H(\cdot) < 1$	CV_w decreases	CV_w increases

Given too many variables in $H(\cdot)$, its behaviour is not amenable to more or less comprehensive formal analysis. It is possible for some particular cases only. For instance, if both y_i and y_k are less than the weighted average and $n_i > n_k$, then $H(\cdot) < 1$ knowingly holds and CV_w diminishes.

In principle, the case of $\Delta CV_w^2 = 0$ is possible as well. Let all regions except i and k have the same per capita income y_r. Then we can aggregate them into a single 'region' r with income y_r and weight $n_r = 1 - (n_i + n_k)$. (Such a 'region' will be used elsewhere below.) In this instance $H(\cdot) = F(y_i, y_k, n_i, n_k; y_r)$. Keeping all variables except y_r constant, we can find the value of y_r such that $H(y_r) = 1$. Equation $H(y_r) = 1$ is a cubic one with respect to y_r; its closed-form solution is very cumbersome and therefore is not reported. (In fact, we can dispense with it, solving the equation numerically.) This equation may have a real positive root, albeit not always. However, no significance should be attached to this fact. First, probability of finding an actual cross-region income distribution (along with the population distribution) that satisfies $H(\cdot) = 1$ even for some single pair of i and k seems to be close to zero. Second, particular cases of satisfying the anonymity principle do not matter at all, while the only (non-degenerate) case – even a single numerical example – of its violation would evidence that the inequality index under consideration does have this unpleasant property.

Table 7 provides numerical examples that illustrate four cases listed in Table 4 and the case of no change in the population-weighted coefficient of variation. It tabulates three income distributions and their permutations – (A), (B) and (C), the population distribution $n = (n_j)$ being uniform across these. Although $\Delta\bar{y}_{(w)} > 0$ holds for all three cases of transition from y to y_*, we can also consider reverse transitions from y_* to y, exchanging indices i and k. In these transitions, $\Delta\bar{y}_{(w)} < 0$. Along with the coefficient of variation, the table reports the population-weighted Gini and Theil indices as well as unweighted inequality indices.

Table 7. Permutation-induced changes in the population-weighted inequality indices.

Region index	n	(A)		(B)		(C)	
		y	y_*	y	y_*	y	y_*
i	0.15	150	300	150	300	150	300
k	0.05	300	150	300	150	300	150
r	0.80	400	400	100	100	218.9	218.9
$\bar{y}_{(w)}$		357.5	372.5	117.5	132.5	212.62	227.62
CV_w		0.251	0.167	0.387	0.537	0.149	0.149
G_w		0.098	0.062	0.129	0.205	0.059	0.060
Th_w		0.039	0.017	0.057	0.115	0.011	0.011
CV		0.363		0.464		0.275	
G		0.196		0.242		0.149	
Th		0.070		0.104		0.038	

Case (A) is that of diminishing values of the population-weighted inequality measures caused by the exchange of incomes between two regions; $H(\cdot) < 1$ here. The decrease is fairly sizeable, equalling more than one third for CV_w and more than a half for Th_w. Considering the reverse transition, we have $\Delta\bar{y}_{(w)} < 0$ and $H(\cdot) > 1$; the permutation of regional incomes causes the weighted inequality indices to rise. In case (B), the effect of permutation in y is an increase in the weighted indices, as $H(\cdot) > 1$; the reverse permutation has the adverse effect. At last, the permutation does not change the weighted coefficient of variation in case (C). Interestingly, the weighted Gini and Theil indices are also near-invariant in this case: $\Delta G_w = 3.7{\cdot}10^{-4}$ and

$\Delta Th_w = 5.5 \cdot 10^{-4}$.

The indices under consideration range from 0 (perfect equality) to some index-specific maximum (perfect inequality). Perfect inequality implies that income is nonzero in a sole region, say, *k*. The second column of Table 8 lists the maxima of *CV*, *G* and *Th*. They depend on the number of country's regions, *m*, only. The violation of the anonymity principle by the population-weighted inequality indices has a crucial corollary: they have no unambiguous maxima. The value taken on by such an index in the case of perfect inequality depends on which specific region *k* possesses all country's income. The relevant maxima are listed in the third column of Table 8.

Table 8. Maxima of unweighted and weighted inequality indices.

Index	Unweighted	Population-weighted
Coefficient of variation	$\sqrt{m-1}$	$\sqrt{1/n_k - 1}$
Gini index	$(m-1)/m$	$1 - n_k$
Theil index	$\log(m)$	$\log(1/n_k)$

The variability of the upper bounds of inequality indices matters in at least two cases. First, to judge how great inequality is from an estimate obtained, we should know how far it is from perfect inequality. Therefore it would be desirable to standardize inequality indices, i.e., to normalize them to their maxima so that they range from 0 to 1 (the Gini index needs such normalization only if the number of regions is small, when $(m-1)/m$ is not sufficiently close to 1). Second, differences in ranges of inequality indices make inequalities incomparable across countries. Lessmann (2014, p. 37) notes that the Theil index is not applicable for cross-country comparison for this reason. However, as it is seen from Table 7, this all the more holds for the coefficient of variation. For example, Williamson's (1965) results are not comparable across countries, as the number of regions varies in his sample from 6 to 75. Thus, the respective maxima of *CV* differ by the factor of more than 3.8. The normalization of inequality indices would solve this problem.

However, Theil (1967, p. 92) objects to normalization, giving an example of two situations. The first society consists of two individuals, only one of them having nonzero income; in the second society, all income belongs to the only of two million persons. The second society is evidently much more unequal. Nonetheless, considerations of cross-country comparability and uniform 'benchmark' of perfect inequality seem more important than Theil's argument (the more so as the number of regions does not differ that dramatically across countries).

In the case of the population-weighted indices, the normalization turns out ambiguous. We could take the 'maximum of maxima', assigning k to the least populated region. (It is such maxima that have been used to standardize the population-weighted indices in Tables 3 and 5.) All the same, this 'global maximum' would depend on the cross-region distribution of country's population. Then the values of a weighted inequality index are not comparable even between countries with the equal number of regions. Moreover, such 'benchmark' of perfect inequality may vary over time in the same country with varying n_k (or even k, if some other region becomes the least populated one).

Let us turn to the principle of transfers which "is usually taken to be indispensable in most of the inequality literature" (Cowell, 2000, p. 98). Let cross-region income distribution $y = (\ldots, y_i, \ldots, y_k, \ldots)$ be transformed into $y_* = (\ldots, y_{*i} = y_i + \theta, \ldots, y_{*k} = y_k - \theta, \ldots)$, where $y_{*j} = y_j$ for $j \neq i, k$, and $0 < \theta < \theta_{\max} = (y_k - y_i)/2$, thus keeping region k still richer than i. Then the weighted average changes: $\Delta\bar{y}_{(w)} = \bar{y}_{*(w)} - \bar{y}_{(w)} = (n_i - n_k)\theta$. The principle of transfers requires the inequality index to decrease under such a transformation. This requirement for the weighted coefficient of variation (denoting $CV_{w*} \equiv CV_w(y_*)$) can be represented as

$$\frac{dCV_{w*}}{d\theta} = \frac{1}{\bar{y}_{*(w)}CV_{w*}}\left(\frac{n_i y_{*i} - n_k y_{*k}}{\bar{y}_{*(w)}} - (CV_{w*}^2 + 1)(n_i - n_k)\right) < 0\,. \tag{10}$$

Condition (10) unambiguously holds only if $n_i y_{*i} < n_k y_{*k}$ and $n_i > n_k$, as both summands in the right-hand side of the equation have negative sign. However, as θ rises, y_{*i} and y_{*k} become progressively closer to each other, which inevitably causes $n_i y_{*i} - n_k y_{*k}$ to change its sign to positive. When the signs of summands in the right-hand side of Equation (10) are different (in the case of $n_i < n_k$ they always are), the resulting sign of their sum depends on particular combination of y, n and the value of θ. Then it is not inconceivable that the derivative of CV_{w*} is positive somewhere in the definitional domain of θ, so violating the principle of transfers.

To show that $dCV_{w*}/d\theta > 0$ is possible, consider the case when the transfer is close to the right bound of its domain, $\theta \approx (y_k - y_i)/2$. Then $y_{*i} \approx y_{*k} \approx (y_k + y_i)/2$. In this instance, provided that $n_i > n_k$, $dCV_{w*}/d\theta > 0$ if $(y_i + y_k)/2 > (CV_{w*}^2 + 1)\bar{y}_{*(w)}$. Let $y_i = (1+\alpha)\bar{y}_{*(w)}$ and $y_k = (1+\beta)\bar{y}_{*(w)}$ (note that α may be negative), then the latter inequality looks like $(\alpha + \beta)/2 > CV_{w*}^2$. Such a relationship is fairly realistic. Usually $CV_{w*} < 1$, therefore α and β

should not be too great. For example, if $CV_{w*} = 0.7$, the principle of transfers will be violated with, say, $y_i = 1.2\bar{y}_{*(w)}$ and $y_k = 1.8\bar{y}_{*(w)}$ in the neighbourhood of $\theta = 0.3\bar{y}_{*(w)}$, or with $y_i = 0.9\bar{y}_{*(w)}$ and $y_k = 2.1\bar{y}_{*(w)}$ near $\theta = 0.6\bar{y}_{*(w)}$. Note that with $n_i > n_k$, a necessary condition for $dCV_{w*}/d\theta > 0$ is exceedance of the weighted average by y_k,

$y_k > \bar{y}_{*(w)} = \bar{y}_{(w)} + (n_i - n_k)\theta > \bar{y}_{(w)}$.

Provided that $n_i < n_k$, $dCV_{w*}/d\theta > 0$ if $(y_i + y_k)/2 < (CV_{w*}^2 + 1)\bar{y}_{*(w)}$. This inequality obviously holds when both y_i and y_k are below the weighted average $\bar{y}_{*(w)}$, or when $\alpha \le -\beta$. It also may be true if both variables are above $\bar{y}_{*(w)}$, e.g. with $y_i = 1.1\bar{y}_{*(w)}$ and $y_k = 1.8\bar{y}_{*(w)}$ near $\theta = 0.35\bar{y}_{*(w)}$, given that $CV_{w*} = 0.7$.

Considering CV_{w*} as a function of transfer, $CV_w(y_*) = CV_w(\theta)$ (then $CV_w(y) = CV_w(0)$), we can distinguish four types of its behaviour (depending on particular y and n). They are depicted in Figure 5, with $CV_w(\theta)$ normalized to $CV_w(0)$ and θ normalized to θ_{max}.

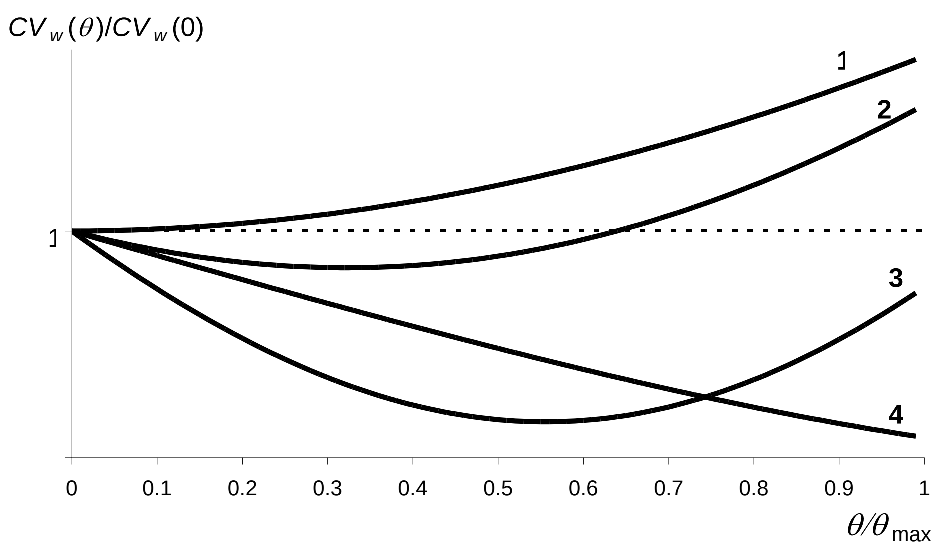


Figure 5. Different types of behaviour of $CV_w(\theta)$.
Note: for all curves, n = (0.15, 0.05, 0,8); for curve 1, $y_{(1)} = (100 + \theta, 300 - \theta, 420)$; for curve 2, $y_{(2)} = (100 + \theta, 300 - \theta, 350)$; for curve 3, $y_{(3)} = (100 + \theta, 300 - \theta, 300)$; for curve 4, $y_{(4)} = (100 + \theta, 300 - \theta, 30)$.

Type 1 is a monotonic rise in the weighted coefficient of variation everywhere in the definitional domain of θ. In type 2, $CV_w(\theta)$ decreases at first and then rises (i.e. $dCV_{w*}/d\theta$ changes its sign from negative to positive). Starting with some θ, it reaches the initial value,

$CV_w(0)$, and then exceeds it more and more. Type 3 is qualitatively similar to type 2, except for $CV_w(\theta)$ does not reach the initial value by the end of the domain of θ. At last, type 4 is a monotonic decreasing $CV_w(\theta)$.

The weighted Gini and Theil indices have the same four types of behaviour. A peculiarity of the Gini index is a break on curve $G_w(\theta)$ in some point (instead of a smooth inflection) in the case of behaviour of types 2 and 3. However, given the same y and n, $G_w(\theta)$ and $Th_w(\theta)$ may differ from $CV_w(\theta)$ in the type of behaviour. For instance, curves of the weighted Gini index corresponding to $y_{(1)}$, $y_{(2)}$ and $y_{(3)}$ in Figure 5 behave according to type 1; the behaviour is similar to that of $CV_w(\theta)$ only in the case of $y_{(4)}$. Curves of the weighted Theil index corresponding to $y_{(2)}$, $y_{(3)}$ and $y_{(4)}$ in Figure 5 have the same type of behaviour as $CV_w(\theta)$, while behaviour of type 2 corresponds to $y_{(1)}$.

The violations of the principle of transfers have important implications for empirical studies. Let we study the evolution of income inequality in some country (assume that the population distribution remains invariant). Provided that the behaviour of the population-weighted inequality measure is of type 1, we would observe increasing inequality with income gaps between regions of the country becoming progressively smaller over time. In the case of behaviour of types 2 and 3, the results will appear even more striking and unaccountable. At first, inequality falls with decreasing income gaps, as could be expected; but then from some point on, further decrease in the income gaps leads to rise in inequality.

Certainly, the situation is much more involved in actual empirical studies. For example, the population-weighted inequality measure may have varied types of behaviour for different region pairs (i, k); besides, an increase in per capita income in the poorer region of a pair is not equal, as a rule, to decrease in the richer region. But the above results evidence that in any case these features of the population-weighted inequality measures will produce (unpredictable) distortions in the pattern of the evolution of inequality.

Usually (albeit not always), dynamics of inequality obtained with the use of different unweighted inequality measures, say, the coefficient of variation, Gini and Theil indices, is qualitatively similar, having the same directions of change in inequality and their turning points. Since different population-weighted indices computed on the same data may have different types of behaviour, they can provide quite diverse patterns of the evolution of inequality in a country, depending on a particular index applied.

Table 9 gives numerical examples of violating the transfer principle for cases (A) $n_i < n_k$

and (B) $n_i > n_k$. It tabulates results for the baseline distribution y and its transformations $y_*(\theta)$ with $\theta = 10$ and $\theta = 90$ ($\theta_{max} = 100$).

Table 9. Transfer-induced changes in the population-weighted inequality indices.

Region	(A)				(B)			
index	n	y	$y_*(10)$	$y_*(90)$	n	y	$y_*(10)$	$y_*(90)$
i	0.05	100	110	190	0.18	100	110	190
k	0.15	300	290	210	0.02	300	290	210
r	0.80	370	370	370	0.80	110	110	110
$\bar{y}_{(w)}$		346.0	345.0	337.0		112.0	120.0	126.4
CV_w		0.178	0.177	0.196		0.242	0.222	0.260
G_w		0.060	0.062	0.079		0.046	0.031	0.104
Th_w		0.021	0.020	0.022		0.020	0.017	0.030
CV		0.446	0.424	0.314		0.541	0.499	0.254
G		0.234	0.225	0.156		0.261	0.235	0.131
Th		0.114	0.101	0.047		0.136	0.116	0.035

In case (A), the population-weighted coefficient of variation and Theil index have behaviour of type 2. Their values decrease with the small transfer $\theta = 10$ and increase with the greater transfer $\theta = 90$. The weighted Gini index behaves according to type 1, its value rising with both transfers. In case (B), all three weighted indices have behaviour of type 2, falling with $\theta = 10$ and rising with $\theta = 90$. Figure 6 illustrates this case graphically for the whole domain of θ.

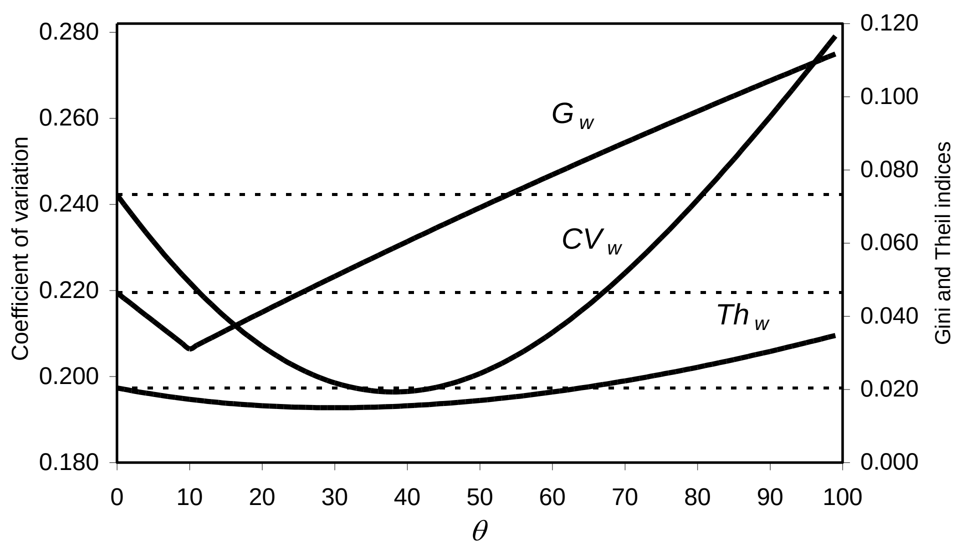


Figure 6. Population-weighted indices as functions of transfer.
Note: the dashed lines correspond to initial levels (with $\theta = 0$) of the indices.

Transfers apart, let us consider case (B) in Table 6 as a pattern of income evolution over three periods, $t = 0, 1, 2$: $y = y_0$, $y_*(10) = y_1$ and $y_*(10) = y_2$. Cross-region income distribution y_1 is evidently more even than y_0; the poorest and richer regions converge to each other; both weighted and unweghted indices indicate diminishing inequality. Then convergence continues; these regions become further closer to each other in y_2. The unveighted indices show further decrease in inequality. However, the weighted indices rise (becoming even greater than for y_0), indicating divergence. The same takes place in similarly interpreted case (A). Thus, contrary to Ezcurra & Rodríguez-Pose's (2009, p. 332) assertion, it is the weighted inequality measures, and not the unweighted ones, that may lead to unrealistic results, affecting our perception of convergence or divergence.

5. Contras and Pros

Williamson's approach to measuring regional inequality did receive some criticism in the literature. Metwally & Jensen (1973) point out:

> Williamson's coefficient […] fails to take into account either the dispersion of incomes nationally, or what is more important in a spatial context, the dispersion of incomes within regions. […] It is possible for this coefficient to decrease over time, suggesting a convergence in regional mean incomes, while dispersion in actual incomes could show an opposite trend. (Metwally & Jensen, 1973, p. 135)

As it is seen, the authors mean measuring national (interpersonal) inequality; therefore their criticism is beside the point. But Williamson (1965) in no way intended to estimate inequality among countries' populations. There is not a grain of evidence of such purpose in his paper; quite the contrary, he highlights throughout the paper that he deals with regional inequality.

Fisch (1984) raises a similar objection:

> Williamson's coefficients of variations ignore a […] critical issue in relation to spatial inequality: the unequal regional distribution of population by income class. (Fisch, 1984, p. 91)

Again, the case in point is inability of the population-weighted coefficient of variation to adequately approximate interpersonal income inequality in the whole country.

In fact, objections due to Metwally & Jensen (1973) and Fisch (1984) are not those to the population weighting. The essence is in that they believe the national inequality rather than regional one to be more proper for Williamson's (1965) research.

Parr (1974) considers a different aspect; he notes:

> [T]he value of the [Williamson] index is likely to be influenced by the regionalization scheme employed, and there will be a tendency for the value of the index to be high when the regionalization involves a relatively large number of regions. (Parr, 1974, p. 84)

This is so indeed concerning the unweighted coefficient of variation with its maximum rising as the square root of the number of regions (see Table 8), but it is not true for the population-weighted index in the general case (as it has been shown in the previous section). The further Parr's note is connected with the weighted index though:

> [T]here is no way of knowing whether the official statistical regions on which the index is based reflect the extent of spatial income differentiation, given the particular number of regions involved. (Parr, 1974, p. 84)

To manage with this problem, the author suggests a bootstrap procedure of placing a number of points, corresponding to the number of official regions, at random over the territory of the country, thus obtaining a standard of spatial income differentiation against which the original index could be compared. It is not entirely clear what Parr means, but it seems that this procedure would yield something like an approximation of the maximum of $\sqrt{1/n_i - 1}$.

Thus, the above considerations do not concern the main sin of the population-weighted indices, their failure in providing unbiased estimates of regional inequality (as well as their unpleasant properties as inequality measures at all). It is not inconceivable that such criticism exists somewhere in the literature; however, I failed in finding it.

Let us turn to arguments in favour of the weighting inequality indices by population. Almost all of them are based on intuitive considerations, being in fact extended versions of Williamson's (1965) statement cited in the Introduction. Lessmann's (2014) reasoning is typical for arguments of such kind:

> [The unweighted inequality measures] cannot account for the heterogeneity of regions with respect to (population) size. This is a very important issue […] due to the lack of a uniform territorial classification for all countries […]. In countries with large economic differences and a very unequally distributed population, an unweighted inequality measure might be difficult to interpret. An example should illustrate the problem. The

northern Canadian Territories are much poorer than the provinces to the south, so that an inequality measure might indicate large economic differences, although very few people are actually poor (note that the Territories are inhabited by only 100,000 people in total). (Lessmann, 2014, p. 37)

The example in this quotation evidently relates to inequality of the whole population of Canada, and not to inequality between regions. Indeed, adding ‘very few people’ living in the Canadian territories to the huge population of the rest of Canada, overall inequality changes only slightly. But this does not imply that there is no inequality between ‘average’ inhabitants of the Territories and the provinces to the south. An analogy may be, e.g., earnings inequality between generals/admirals and other military personnel in the US Armed Forces. ‘Per capita’ salary of the former is al least three times higher than that of the latter, which implies rather high inequality. Comparing these ‘per capita’ salaries, why should we care about the percentage of generals/admirals in the Armed Forces? Provided that this percentage is very small, 0.069% (Kapp, 2016, p. 5), the value of any weighted inequality index will be close to zero, thus suggesting no (significant) inequality between generals/admirals and other servicemen. The example of Canada is worth considering in more detail with the use of actual data reported in Table 10.

Table 10. Personal income and population in Canada, 2013.

Region	Total income, million Canadian dollars [1]	Population, thousand people [2]	Income per capita, Canadian dollars	Region weight	Region weigh among provinces
Canada	1,222,216	35,102	34,819		
Provinces	1,217,972	34,986	34,813	0.997	
Newfoundland and Labrador	18,027	528	34,158	0.015	0.015
Prince Edward Island	4,241	145	29,228	0.004	0.004
Nova Scotia	29,378	944	31,125	0.027	0.027
New Brunswick	22,693	756	30,025	0.022	0.022
Quebec	257,579	8,144	31,626	0.232	0.233
Ontario	468,655	13,538	34,618	0.386	0.387
Manitoba	38,445	1,264	30,419	0.036	0.036
Saskatchewan	39,114	1,102	35,487	0.031	0.032
Alberta	181,359	3,979	45,577	0.113	0.114
British Columbia	158,481	4,586	34,556	0.131	0.131
Territories	4,244	115	36,832	0.003	
Northwest Territories	1,816	36	50,160	0.001	
Yukon	1,439	44	32,890	0.001	
Nunavut	989	35	28,042	0.001	

[1] Canada Revenue Agency (2016). Returns from outside Canada are excluded.
[2] Statistics Canada (2016a). Annual average (the arithmetic mean of quarterly estimates).

Population of three Canadian territories comprise only 0,33% of the total country's population. Here are the poorest and richest regions of Canada, the difference in incomes per capita between them equalling 79%. As compared with the richest region among provinces, Alberta, income per capita there is 63% higher than in Nunavut. Contributing to the total population one order of magnitude smaller than the Nenets AO in the example from Section 3 (see Table 4), all three territories much less are able to change the overall income inequality in the country. This notwithstanding, the average inhabitant of Nunavut remains 1.6 times poorer than the average inhabitant of Alberta. It does not matter a hoot for this fact that they are only 35 thousand in number.

Table 11 presents estimates of inequality measures in two spatial samples: for all Canadian regions and for provinces only (i.e., excluding the northern territories). Note that only standardized values are comparable across the samples, since these differ in the number of regions as well as in the least populated regions; the normalizing factor for CV_w, $1/\sqrt{1/n_k - 1}$, equals 0.032 for all Canadian regions and 0.065 for provinces only.

Table 11. Estimates of income inequality in Canada.

Index	Unweighted		Population-weighted	
	Raw	Standardized	Raw	Standardized
	All regions			
Coefficient of variation	0.180	0.052	0.120	0.004
Gini index	0.089	0.096	0.054	0.054
Theil index	0.015	0.006	0.007	0.001
	Provinces only			
Coefficient of variation	0.133	0.044	0.119	0.008
Gini index	0.065	0.072	0.054	0.054
Theil index	0.008	0.004	0.007	0.001

The unweighted indices indicate a decrease in inequality between regions when the territories are deleted from the spatial sample. It is quite understandable, as both the richest and poorest regions are excluded. The weighted Gini (as well as Theil) index remains invariant, as if measuring overall population's inequality But it is by no means close to the Canadian Gini index for 2013 equalling 0.358 (Statistics Canada, 2016b). Thus the weighted Gini index considerably underestimates both regional and interpersonal inequalities. The (standardized) weighted coefficient of variation behaves strikingly; it doubles when the northern territories are eliminated, suggesting a rise in inequality. Then what is difficult to interpret, the unweighted or weighted coefficient of variation?

Gisbert (2003) gives the reason for the weighting by population in a kernel density of cross-country income distribution, based on reasoning essentially similar to that in Lessmann (2014), albeit in the international context. As he points out,

> [Unweighted kernel density] abstracts from the ‘size’ of the different countries. […] [T]he world income distribution in terms of countries […] can be highly misleading, for example if we drew national borders differently this would affect the shape of the densities […]. The natural alternative is to attach a weight to the observations where the weights reflects the contribution of each observation in the sample. In our example, per capita GDP, the obvious weight is the population (POB) of each country. […] [P]opulation is very unevenly distributed among countries; for example China and India, two of the poorest countries, account for more than one third of the total population in the world, on the other side some of the richest countries, like Iceland or Luxembourg, only account for 0.01% of the world population. It does not seem fair to treat all these countries equally in estimation. (Gisbert, 2003, p. 337–338)

This reasoning again relates to the whole population, this time, of the world. Returning to the example of the US Armed Forces, let us draw the ‘border’ in such a way as to add colonels/navy captains to generals/admirals. Certainly, the ‘cross-rank’ earnings distribution as well as inequality between this group and the group of other servicemen changes, while the earning distribution and inequality in the whole US Armed Forces remains intact. However, the case at hands is two different phenomena, first, inequality between an (‘average’) high-rank officer and (‘average’) serviceman with no high rank and the relevant earnings distribution, and second, inequality in the whole military personnel and cross-person earnings distribution in the US army.

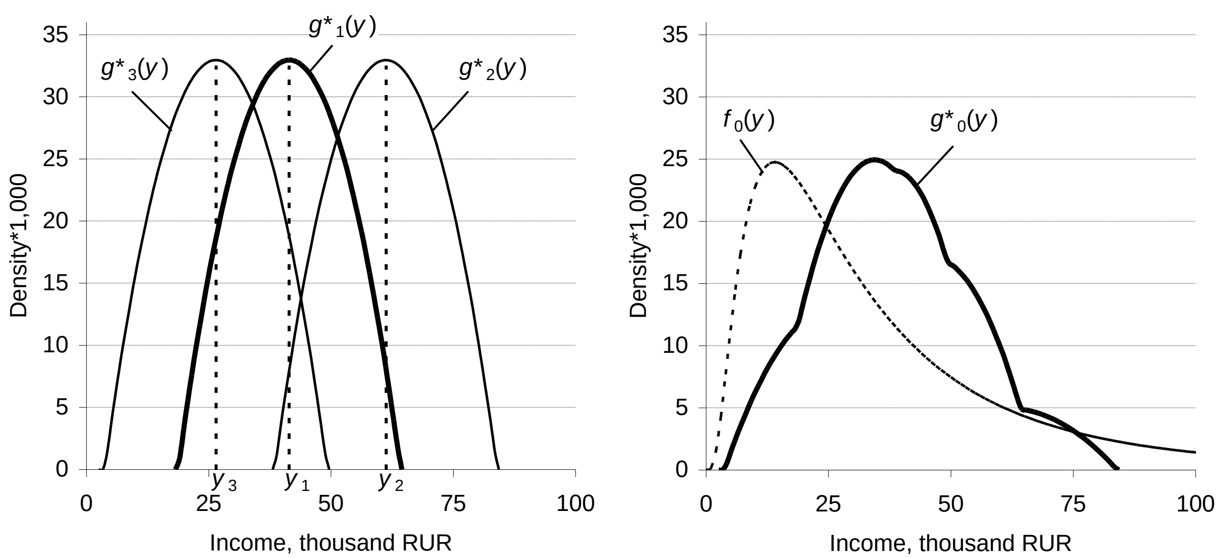
The kernel density estimator is a standard way of constructing continuous distribution from a set of *m* discrete observations (Silverman, 1986). It is defined as (in notation of this paper):

$$f(y) = \frac{1}{m}\sum_{i=1}^{m} K(\frac{y - y_i}{h}) / h, \tag{11}$$

where *h* is the smoothing bandwidth which depends on the number of observations, *m*, as well as on parameters of the source distribution (y_i); $K(\cdot)$ is a kernel function. Considering regions instead of countries, $f(y)$ is an estimate of cross-region income distribution (to be exact, the probability density). Based on his reasoning, Gisbert (2003, p. 338) modifies Formula (11) in the following way:

$$g_0^*(y) = \sum_{i=1}^{m} n_i K(\frac{y - y_i}{h}) / h \equiv \sum_{i=1}^{m} n_i g_i^*(y) . \qquad (12)$$

This formula resembles Formula (7) in Section 3. The similarity is not formal; the essence of Formulae (7) and (12) is the same. Both approximate cross-person income distribution in the whole territory consisting of m territorial units. The difference is in that the delta function representing within-region income distribution $g_i(y)$ in Formula (7) is replaced by an arbitrary – with respect to the actual within-region distribution – function $g_i^*(y)$ in Formula (12). A number of functions can serve as the kernel in Formulae (11) and (12). To be specific, employ the Epanechnikov kernel: $K(\frac{y - y_i}{h}) = K_i^E(y) = 0.75(1 - \frac{(y - y_i)^2}{h^2})$ if $y \in [y_i + h, y_i - h]$, otherwise $K_i^E(y) = 0$. Then $g_i^*(y) = K_i^E(y)/h$. Figure 7 shows such proxies of the regional income distributions (in the left panel) and income distribution of the whole population (in the right panel) as applied to the example of the Tyumen Oblast from Section 3.



(a) Subregions of the Tyumen Oblast

(b) The Tyumen Oblast as a whole

Figure 7. Proxies of income distributions in the Tyumen Oblasts.
Note: $f_0(y)$ is drawn from Figure 4 (d).

In this example, $h = 0.9 \cdot (4\pi)^{0.1} \cdot (15/m)^{-0.2} \cdot \sigma = 22.756$, where σ = standard deviation of y_is. It artificially 'imputes' income dispersion to regions; as a result, $CV(g_0^*(y))$, $G(g_0^*(y))$ and $Th(g_0^*(y))$ are not equal to CV_w, G_w and Th_w, respectively. Nonetheless, the former also do not provide adequate estimates of population's income inequality. Comparing Figure 7 (a) with Figure 4 (c), it is seen that the 'imputed' regional distributions $g_i^*(y)$ are far from being similar to the actual distributions $f_i(y)$. Owing to this, their weighted sum (weighted kernel density estimate) $g_0^*(y)$ is a severely distorted proxy of the actual population income distribution $f_0(y)$ as Figure 7 (b) evidences.

Petrakos et al. (2005, p. 1839–1840) derive the need for the population weighting from a critique of the β-convergence methodology. According to them, analysis of β-convergence can distort the perception of convergence trends, since it neglects relative sizes of regions. To illustrate this statement, the authors offer a simple three-region example. Table 12 tabulates this example (Petrakos et al., 2005, p. 1840), supplementing it with estimates of different inequality measures, both unweighted and population-weighted (with no normalization). Among them, σ stands for the standard deviation of log income and σ_w is its population-weighted counterpart.

Table 12. Inequality estimates in Petrakos' et al. (2005) example.

Region	Population	*n*	*y*(*t*)	*y*(*t*+*τ*), scenario 1	*y*(*t*+*τ*), scenario 2
A	4.0	0.714	20	25	25
B	1.5	0.268	14	15	15
C	0.1	0.018	6	7	8
$\bar{y}_{(w)}$			18.143	22.000	22.018
CV / CV_w			0.430 / 0.172	0.470 / 0.221	0.436 / 0.218
G / G_w			0.233 / 0.075	0.255 / 0.099	0.236 / 0.098
σ / σ_w			0.505 / 0.215	0.523 / 0.271	0.466 / 0.261
Th / Th_w			0.100 / 0.017	0.115 / 0.027	0.097 / 0.026

The initial state, $y(t)$, is compared with a final state, $y(t+\tau)$, under two scenarios. Regarding scenario 1, both β-convergence analysis and all inequality indices unambiguously indicate income divergence. However, β-convergence occurs under scenario 2, while CV_w suggests divergence. Petrakos et al. (2005) assign this to the fact that fast growth of small region C (by 33%) blurs the picture when all regions are treated as equal, whereas CV_w accounts properly for the relative importance of region C and therefore adequately indicates divergence. However, the unweighted indices CV and G also indicate divergence under scenario 2. At the same time, σ and Th suggest convergence. Hence the weighting is not the case; the point is that specific inequality measures differ in sensitivity to changes in income distribution (Lambert, 2001). As for β-convergence, it results from diminishing σ under scenario 2. Wodon & Yitzhaki (2006) prove that from σ-convergence follows β-convergence (but the converse is not true: β-convergence does not necessary implies σ-convergence). All weighted inequality indices, indeed, indicate divergence under scenario 2. However, this is a particular case. For example, if population of region C were 1.0 instead 0.1, σ_w would suggest convergence, being equal to 0.451 in the initial state and 0.439 under scenario 2.

A sole attempt to justify the need for weighting by population through quantitative analysis is due to Portnov & Felsenstein (2010). They explore the sensitivity of four unweighted and population-weighted inequality measures to changes in the ranking, size and number of regions into which a country is divided, explicitly treating regions as groups of people. One of their tests consists in comparison between two situations that differ in the cross-region population distribution and national per capita income, keeping the cross-region income distribution invariant. Surprisingly, the values of the unweighted indices change across the situations, although they should not, being independent of the population distribution. A closer look shows that this is due to the mistaken use of $\bar{y}_{(w)}$ instead of $\bar{y}$ in calculation of these indices. In one more test, the population distribution randomly changes, the cross-region income distribution and national per capita income being kept constant. As can be expected, the weighted inequality indices react to these changes, while the unweighted ones remain constant. The authors believe the latter to be a shortcoming. They conclude:

> These [unweighted] indices may thus lead to spurious results when used for small countries, which are often characterized by rapid changes in population patterns. (Portnov & Felsenstein, 2010, p. 217)

They also conclude that the population-weighted indices – the Williamson coefficient of variation, Gini index and Coulter coefficient – may be considered as more or less reliable regional inequality measures (Portnov & Felsenstein, 2010, pp. 217–218). Both conclusions are fallacious. Explicitly treating regions as groups of people, the authors implicitly deal with the estimation of interpersonal inequality in the country, misinterpreting it as the estimation of regional inequality. Therefore, their results in no way can be deemed a proof of the use of weighting.

The above considerations show that supporters of the weighting by population confuse inequality between regions (i.e. between representative inhabitants of regions) and the overall interpersonal inequality. According to them, the population weights should reflect the contribution of each territorial unit. But, contribution to what? They interpret it as a contribution to inequality between regions, while in fact it is a (proxy of) contribution to inequality between all inhabitants of the set of territorial units under consideration.

Studies on international inequality also widely use the population-weighted indices. From all appearances, economists engaged in studies of international inequality ‘reinvented’ Williamson’s approach. In contrast to regional researchers (who sometimes perform international studies as well), they are aware of the conceptual distinction between unweighted and population-weighted inequality indices, explicitly interpreting the latter as approximate measures of inequality among the world population, and not between nations. A surprising thing is that as if there is a barrier between the literature on regional inequality and that on international inequality. The former almost never references to the latter (Akita et al., 2011, can be mentioned as one of extremely rare examples). The conversance with the literature on international inequality would surely prevent regional researchers from misinterpreting the population-weighted indices as measures of regional inequality.

While the literature on regional inequality does not discuss need for the population weighting in inequality indices, getting by short notes like those cited throughout this paper, the literature on international inequality widely debates the question ‘To weight or not to weight?’. Both viewpoints are considered in detail by e.g. Firebaugh (2003) and Ravallion (2005). Under interpretation of the population-weighted estimates as proxies of inequality among the world population, the arguments in favour of weighting look reasonable; at least, they are seriously substantiated.

However, the results of applying the population-weighted indices for estimation of global inequality are disappointing as, e.g., the results due to Milanovic (2012) cited in the end of Section 2 suggest. This is of no surprise in the light of the above exposition. As Milanovic (2005, p. 10) notices, population-weighted inequality “deals neither only with nations nor individuals but falls somewhere in between”. (In fact, this is not always true; it may fall below the both as examples of Russia in Table 5 and Canada in Table 11 evidence.) He also accepts that it may be misleading (Milanovic, 2012, p. 8). Worse yet, this is the prevailing situation as it is proved in Section 3: estimates of interpersonal inequality with the use of the population weighting are always severely distorted.

The debate regarding the population weighting in the literature on international inequality focuses on the issue of what an adequate characterization of inequality in the world is, either inter-country inequality or interpersonal inequality among the world population. In my view, this debate is fairly pointless. It must be agreed with Firebaugh (2003), who notes that the answer depends on the goal:

> [T]he issue of unweighted versus weighted between-nation inequality reduces to this question: are we interested in between-nation income inequality because of what it tells us about the average difference between nations' income ratios, or because of what it tells us about the average difference between individuals' income ratios? (Firebaugh, 2003, p. 129)

At last, one more issue needs to be touched upon. Exploring determinants of regional inequality with the use of population-weighted inequality indices, some authors, e.g. Kyriacou & Roca-Sagalés (2014) and Lessmann (2014), also employ unweighted indices for robustness checks. Such a way seems contradictory. On the one hand, if the authors believe unweighted measures to distort perception of inequality, then why should these measures confirm results obtained with the use of 'adequate' measures? On the other hand, if they do confirm, then why do we need the weighting?

6. Conclusions

Following Williamson (1965), many economists estimate regional inequality with the use of indices weighted by regions' shares in the national population. Analysis in this paper shows that this approach is conceptually inconsistent. Instead of an estimate of regional inequality, we get a rough estimate of interpersonal inequality among the whole population of the country (and this estimate makes sense only if it deals with indicators applicable to an individual). Therefore the population-weighted estimates of inequality are biased with regard to estimates of both regional inequality (as they measure a different value) and interpersonal inequality (as they do not and cannot take account of within-region income disparities). In both cases, the result may be not only distorted, but also quite misleading. Thus the population-weighted inequality indices **never** give adequate results.

Moreover, the population-weighted inequality indices do not satisfy requirements for an adequate inequality measure. They violate three of four basic axioms, the population, anonymity and transfer principles. This may lead to estimates of inequality evolution that contradict common sense. One more consequence is the absence of unambiguous maxima of the population-weighted inequality indices. This makes it impossible to standardize estimates of inequality with the aim of cross-time or cross-country comparability.

Thus, it can be concluded that the application of the population-weighted indices to measuring regional inequality is nothing but a fallacy.

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